

MTH 103 – Algebraic Reasoning

Course Notebook

Name: _____

Authors: Sara Clark, Elizabeth Jones, Lyn Riverstone, Katy Williams

Course Information

Class meetings: _____

Instructor: _____

Office: _____

Office hours: _____

Exam 1: _____

Exam 2: _____

Final Exam: _____

***** *Always log into ALEKS via our Canvas course site.* *****

Getting Help Outside of Class

	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
8am							
9am		MSLC	MSLC	MSLC	MSLC	MSLC	
10am		MSLC	MSLC	MSLC	MSLC	MSLC	
11am		MSLC	MSLC	MSLC	MSLC	MSLC	
12pm		MSLC	MSLC	MSLC	MSLC	MSLC	
1pm		MSLC	MSLC	MSLC	MSLC	MSLC	
2pm		MSLC	MSLC	MSLC	MSLC	MSLC	
3pm		MSLC	MSLC	MSLC	MSLC	MSLC	
4pm		MSLC	MSLC	MSLC	MSLC		
5pm							
6pm							
7pm	MSLC	MSLC	MSLC	MSLC	MSLC		
8pm	MSLC	MSLC	MSLC	MSLC	MSLC		
9pm	MSLC	MSLC	MSLC	MSLC	MSLC		

- Add your Instructor's and your TA's office hours to this weekly schedule!
- MSLC: math tutoring in the Math & Stats Learning Center, Kidder Hall 108 (**tutors available weeks 2-10, not finals week**)

Table of Contents

Chapter 1 – What is a Function?	1
Chapter 2 – What Can We Learn from a Graph?	25
Chapter 3 – How is a Parent Function Related to Other Members of its Family?.....	45
Chapter 4 – How Dow We Model Data?.....	65
Chapter 5 – What Can We Learn from Equations?.....	83

Preface

Welcome to Algebraic Reasoning! In this course you will develop the skills you need to be successful in your College Algebra course. College Algebra satisfies the Baccalaureate Core Mathematics requirement here at Oregon State University. The rationale for this requirement is:

Everyone needs to manipulate numbers, evaluate variability and bias in data (as in advertising claims), and interpret data presented both in numerical and graphical form. Mathematics provides the basis for understanding and analyzing problems of this kind. Mathematics requires careful organization and precise reasoning. It helps develop and strengthen critical thinking skills.

Function Families Approach

Studying functions may seem overwhelming at first; there are an infinite number of them after all! However, we will be systematic in our approach, first carefully examining some basic functions, and categorizing them according to their common characteristics. These categories we call *families* of functions. We study five different families in this course:

1. Linear
2. Quadratic
3. Absolute value
4. Square Root
5. Exponential

Each family has a parent function. Studying these five parent functions in depth, and exploring how each one is related to the other functions in its family, allows us to lay a foundation for understanding *all* functions. As you learn about an entire family, rather than a bunch of individual functions, you may find you won't need to memorize as much as you have in previous mathematics courses.

As you learn about the five function families, you will also build mathematical skills, such as reasoning and communicating using different function representations (graphical, verbal, symbolic, or numerical). For instance, you will practice: sketching quick graphs to help you understand functions; using tables to determine how output values of a function change as the input changes; and moving from a verbal description a mathematical relationship between two quantities, to an equation that represents the relationship.

Focus on Mathematical Modeling

With a deeper understanding of function families, and greater confidence in working with and moving between their various representations, you will be able develop mathematical models for many real-world situations. You will then be able to analyze the function, determine its properties, and interpret these in context to answer questions about the situation being modeled, such as, "How is the value of a smartphone changing over time?"

Features of this Course Notebook

Learning Objectives:

Each chapter in this Course Notebook is divided into multiple Lessons, each of which is organized around a set of learning objectives. The learning objectives are listed at the start of each Lesson.

Team Learning:

In class you will work with your team to solve the problems and answer the questions posed in the notebook. As you work through this course, there are four main types of activities you and your team will be doing: 1) Warm-ups; 2) Lessons; 3) Algebra Critiques; and 4) Wrap-ups.

**Warm-up:**

Most Lessons in the notebook begin with a Warm-Up activity, designed to elicit prerequisite knowledge needed for success on the upcoming Lesson.

**Lesson:**

As you work with your team to solve the problems presented in each Lesson, you will learn new concepts, build understanding and connections, and practice your mathematical reasoning and communication skills.

**Algebra Critique:**

Some lessons include activities designed to help you refine your algebra skills by critiquing student work, explaining what went wrong (if anything), and correcting the error. Many of these critiques include common student mistakes and misconceptions.

**Wrap-up:**

Finally, to check your understanding of the material studied in the Lesson, you will work with your team to complete a Wrap-up activity, to be handed in at the end of class. Many of the Wrap-up problems provide an opportunity to apply what you have learned in the Lesson to real-world contexts.

ALEKS:

In addition to these in-class activities, you will also complete assignments in ALEKS both *before* and *after* class. The preparation assignments are designed to make sure you have the pre-requisite knowledge needed for success with the in-class activities. These ALEKS topics are introductory, and may be a refresher on previously-learned math concepts or procedures. After class, you will work on the more challenging ALEKS topics, to reinforce your in-class learning, practice mathematical procedures, and gauge your understanding to better focus your study efforts.

Let's roll up our sleeves and get started. You've got this!

Chapter Learning Objectives

1. From a verbal description, table, arrow diagram, list of ordered pairs, or graph, determine whether a given relation is a function.
2. State the domain and range of a relation given verbally, in a table, arrow diagram, list of ordered pairs, or graph.
3. Given their graphical, symbolic, or numerical representations, identify each of the five types (or *families*) of functions studied in this course:
 - Linear
 - Quadratic
 - Absolute value
 - Square root
 - Exponential
4. Look for patterns and apply definitions of exponents to develop Properties of Exponents.
5. Apply properties of exponents to simplify or evaluate exponential expressions.
6. For each of our five function families, use a table or equation to evaluate the function for a given input, and express the value using function notation (for example, find $f(-8)$ for $f(x) = x^2$).
7. Explain the difference between $f(x) = 2$ and $f(2)$.
8. Sketch graphs of the five parent functions.
9. Translate between a symbolic representation and a verbal description of a function.
10. Use a graphing tool to explore the graphs of our five function families.

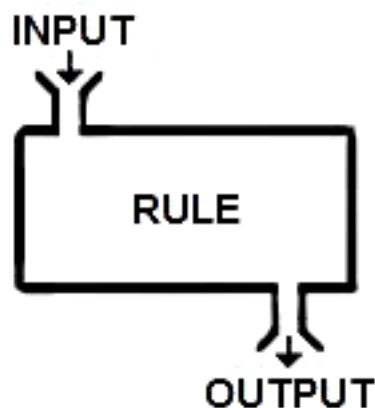
Chapter 1

What is a Function?

Chapter Overview

In Chapter 1, we begin our work with the concept of function. Five families of functions form the basis of this Algebraic Reasoning course: linear, quadratic, square root, absolute value, and exponential.

In this first chapter we introduce our five function families, their symbolic, graphical, and numerical forms, as well as their domains and ranges, and the parent function of each family. We will also become familiar with function notation and the meanings of other mathematical notations we will use throughout this course.



Chapter 1 Contents

Chapter Learning Objectives	1
Chapter Overview	1
1.1: Introduction to Relations & Functions	3
1.2: Families of Functions Reference Guide	7
1.3: Properties of Exponents	13
Properties of Exponents Reference Guide	16
1.4: Algebra Critique – Properties of Exponents.....	17
1.5: Function Notation	19
1.6: Translating Between Equations and Words.....	23



1.1: Introduction to Relations & Functions

Learning Objectives

Together with your team:

- From a verbal description, table, arrow diagram, list of ordered pairs, or graph, determine whether a given relation is a function.
- State the domain and range of a relation given verbally, in a table, arrow diagram, list of ordered pairs, or graph.

1) A **relation** is any set of ordered pairs, (input, output). All of the following representations below show relations between an **input variable** and an **output variable**.

a) In the space provided for each relation, write the set of ordered pairs represented.

<p>Ordered pairs:</p>	<p>Ordered pairs:</p>																				
<p>Ordered pairs:</p>	<p>Ordered pairs:</p>																				
<p>Ordered pairs:</p> <table border="1"> <thead> <tr> <th>Input</th><th>Output</th></tr> </thead> <tbody> <tr> <td>0</td><td>2</td></tr> <tr> <td>2</td><td>7</td></tr> <tr> <td>4</td><td>7</td></tr> <tr> <td>6</td><td>-10</td></tr> </tbody> </table>	Input	Output	0	2	2	7	4	7	6	-10	<p>Ordered pairs:</p> <table border="1"> <thead> <tr> <th>Input</th><th>Output</th></tr> </thead> <tbody> <tr> <td>1</td><td>13</td></tr> <tr> <td>4</td><td>-6</td></tr> <tr> <td>4</td><td>7</td></tr> <tr> <td>9</td><td>10</td></tr> </tbody> </table>	Input	Output	1	13	4	-6	4	7	9	10
Input	Output																				
0	2																				
2	7																				
4	7																				
6	-10																				
Input	Output																				
1	13																				
4	-6																				
4	7																				
9	10																				

b) Each relation in the left column (above) is a **function** and each relation in the right column is a **non-function**. On your own write what you think the difference is between a function and a non-function.

- c) Now, share your answer to b) with your team and come up with a one-sentence summary of the difference between a function and a non-function. Be ready to share with the class.

Definitions we will use for this class:

A **relation** is any set of ordered pairs, $(x, y) = (\text{input}, \text{output})$.

A **function** is:

The **domain** of a relation is:

The **range** of a relation is:

- 2) In the previous problem, we saw several ways to represent a relation: a graph, an arrow diagram, and a table. We can also represent relations using words; that is, by writing a verbal description of the relationship between the inputs and outputs.

For each relation given in a) and b) below, decide whether it is a function. Explain why or why not.

a) A relation inputs an Oregon State Student ID Number and outputs the name of the OSU student.

b) A relation inputs a date (e.g. 01/01/1999) and outputs the name of a person with their birthday on that date.

For each relation given in **3) – 8)** below,

- decide whether the relation defines the output as a *function* of the input, and
- state its domain and range.

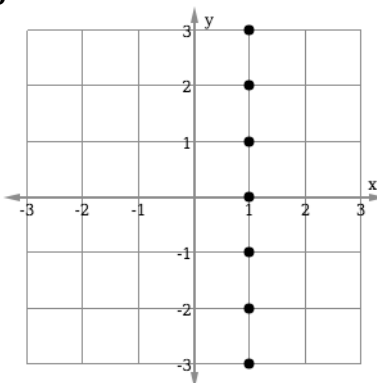
3)

Age (months)	Average Weight of Female Babies (pounds)
0	7.3
1	9.6
2	11.7
3	13.3
4	14.6
5	15.8
6	16.6

Function? YES NO

Domain:

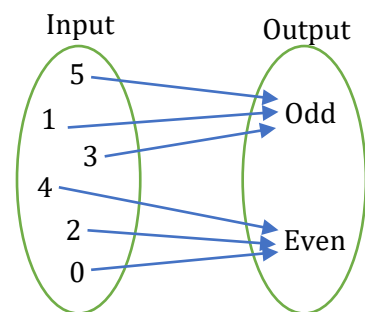
Range:

4)

Function? YES NO

Domain:

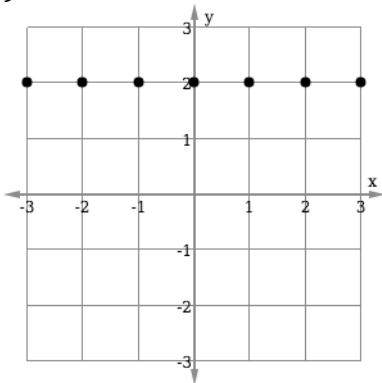
Range:

5)

Function? YES NO

Domain:

Range:

6)

Function? YES NO

Domain:

Range:

7)

$$\{(a, 1), (b, 2), \dots, (z, 26)\}$$

Function? YES NO

Domain:

Range:

8) A relation that takes any real number as input and outputs twice the number.

Function? YES NO

Domain:

Range:

Summary: How to Determine Whether a Relation is a Function

From a graph:

From an arrow diagram:

From a table:

From a list of ordered pairs:

From a verbal description:

- 9) Other than the representations listed in the Summary box above, can you think of another way we commonly represent functions?



1.2: Families of Functions Reference Guide

Learning Objectives

Together with your team:

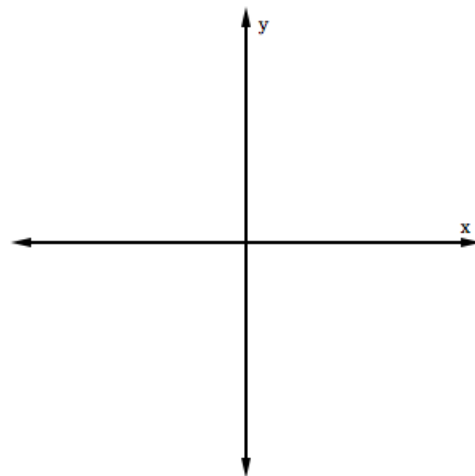
- Identify each of the five families of functions (linear, quadratic, absolute value, square root, exponential) studied in this course, given their graphical, symbolic, or numerical representations.
- Sketch a graph of the five parent functions.
- Use a graphing tool to explore the graphs of our five function families.

We will study Five Function Families in this class. Each function family has a Parent Function.

Function Family	Equation of Parent Function
Linear	
Quadratic	
Absolute value	
Square root	
Exponential	

We will sketch what we like to call “good enough” graphs.

What is a “good enough” graph?



Recall:

- ✓ A **relation** is any set of ordered pairs (x, y) .
- ✓ All the x -values (inputs) in the ordered pairs together make up the **domain** of the relation.
- ✓ All the y -values (outputs) in the ordered pairs together make up the **range** of the relation.
- ✓ A relation is a **function**, if each x -value is paired with *exactly one* y -value.

Directions: Go to the website www.desmos.com and click “Start Graphing.” Graph each of the following parent functions in Desmos and record a sketch of a “good enough” graph of each. Then, describe the shape of the graph.

Linear Family

Parent Function

Symbolic representation (an equation):

$$y = x$$

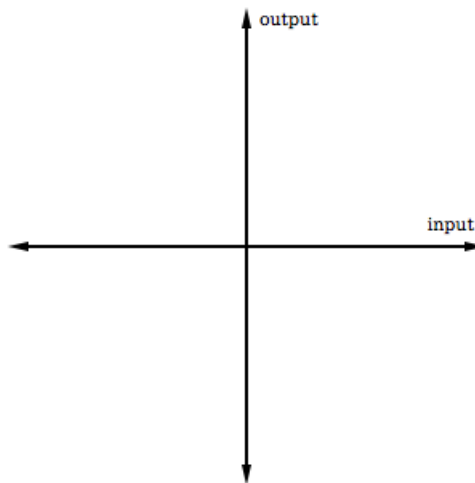
Numerical representation (a table of values):

x	$y = x$
-2	-2
-1	-1
0	0
1	1
2	2

Verbal description:

The value of y is the same as the value of x .

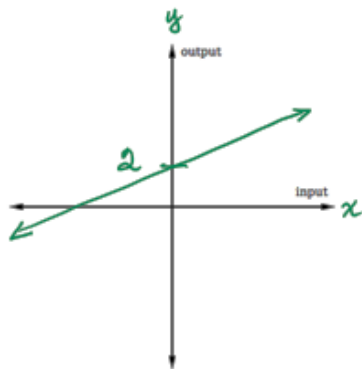
Graphical representation (a graph):



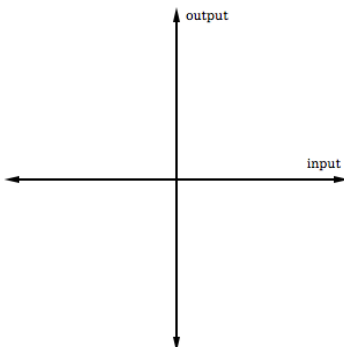
- Graph shape: _____
- Domain of $y = x$: **x is any real number**
- Range of $y = x$: **y is any real number**

Examples of Linear Functions

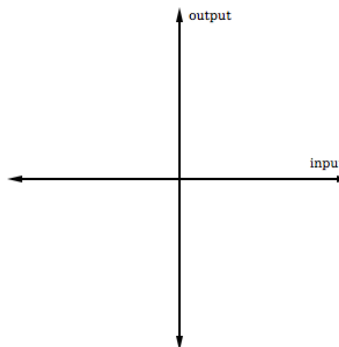
1. $y = \frac{1}{3}x + 2$



2. _____



3. _____



Notes About Linear Family

- Possible number of x -intercepts: 0 1 2 3
- Possible number of y -intercepts: 0 1 2

Quadratic Family

Parent Function

Symbolic representation:

$$y = x^2$$

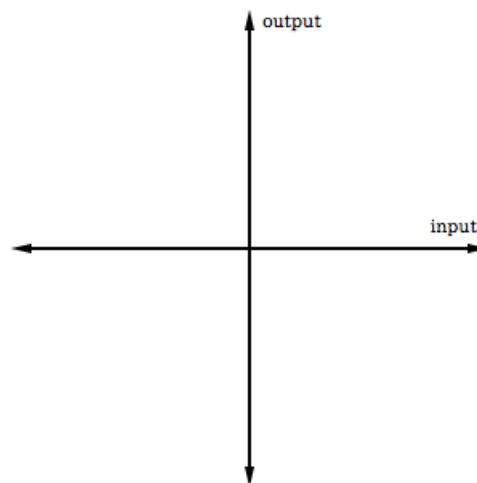
Numerical representation:

x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4

Verbal description:

*The value of y is x **squared**; that is, y is the number we get when we multiply x by itself.*

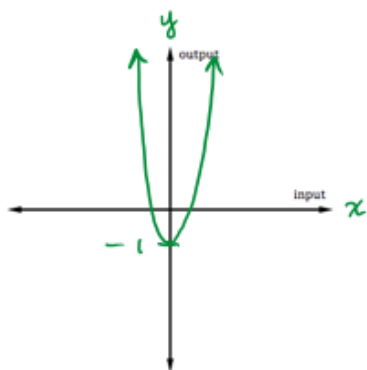
Graphical representation:



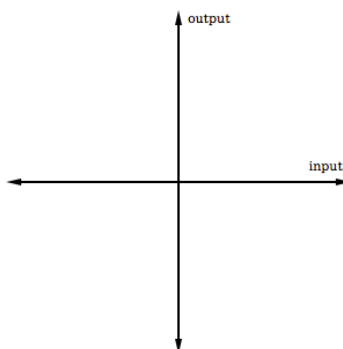
- Graph shape: _____
- Domain of $y = x^2$: **x is any real number**
- Range of $y = x^2$: **$y \geq 0$**

Examples of Quadratic Functions

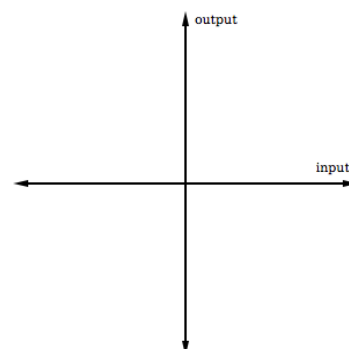
1. $y = 3x^2 - 1$



2. _____



3. _____



Notes About Quadratic Family

- Possible number of x -intercepts: 0 1 2 3
- Possible number of y -intercepts: 0 1 2

Square Root Family

Parent Function

Symbolic representation:

$$y = \sqrt{x}$$

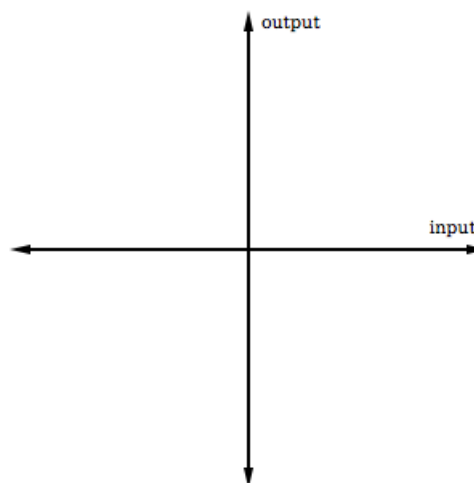
Numerical representation:

x	$y = \sqrt{x}$
-2	undefined
-1	undefined
0	0
1	1
2	$\sqrt{2} \approx 1.41$

Verbal description:

The value of y is the **square root** of the value of x ; that is, y is the number that must be multiplied by itself to get x .

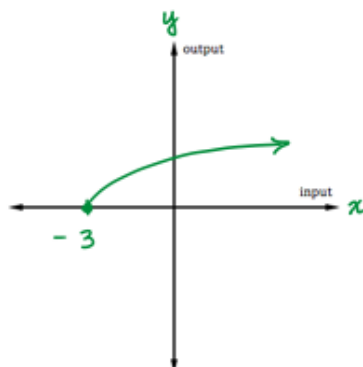
Graphical representation:



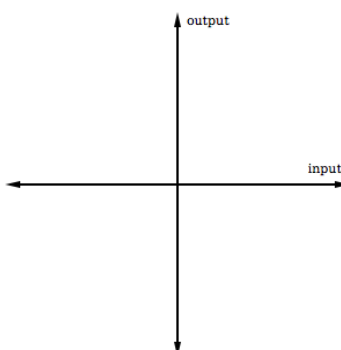
- Graph shape: _____
- Domain of $y = \sqrt{x}$: $x \geq 0$
- Range of $y = \sqrt{x}$: $y \geq 0$

Examples of Square Root Functions

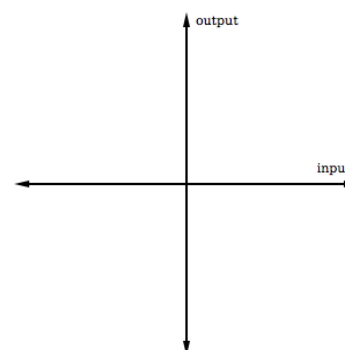
1. $y = \sqrt{x + 3}$



2. _____



3. _____



Notes About Square Root Family

- Possible number of x -intercepts: 0 1 2 3
- Possible number of y -intercepts: 0 1 2

Absolute Value Family

Parent Function

Symbolic representation:

$$y = |x|$$

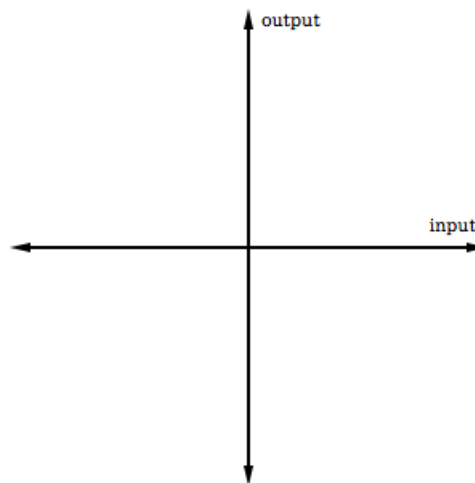
Numerical representation:

x	$y = x $
-2	2
-1	1
0	0
1	1
2	2

Verbal description:

*The value of y is the **absolute value** of x ; that is, y is the distance between x and 0.*

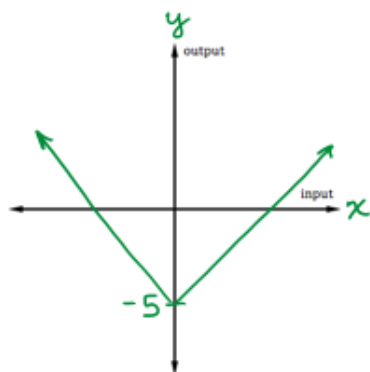
Graphical representation:



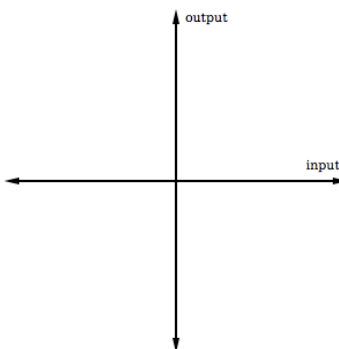
- Graph shape: _____
- Domain of $y = |x|$: **x is any real number**
- Range of $y = |x|$: **$y \geq 0$**

Examples of Absolute Value Functions

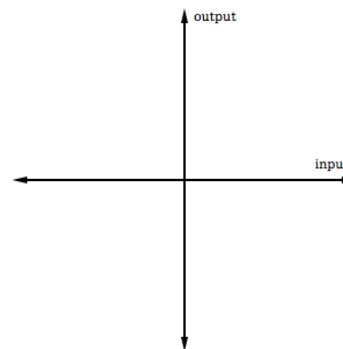
1. $y = |x| - 5$



2. _____



3. _____



Notes about Absolute Value Family

- Possible number of x -intercepts: 0 1 2 3
- Possible number of y -intercepts: 0 1 2

Exponential Family

Parent Function

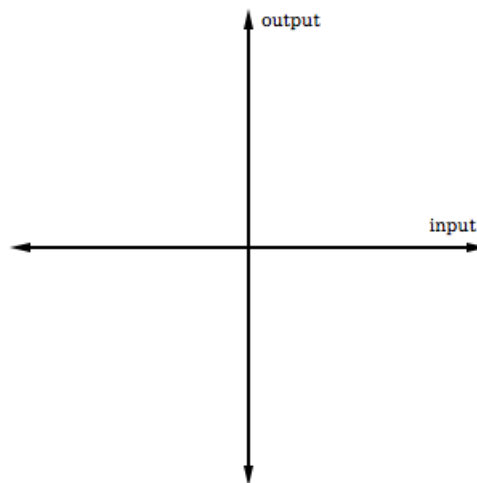
Symbolic representation:

$$y = a^x \text{ with, } a > 0, a \neq 1$$

Numerical representation:

x	$y = 2^x$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Graphical representation:



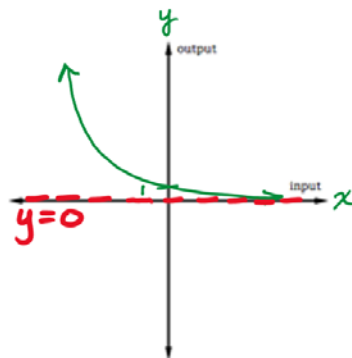
Verbal description:

The value of y is the **base-2 exponential** of x ; that is, y is the number we get when we raise the base (2 in this case) to the power of x .

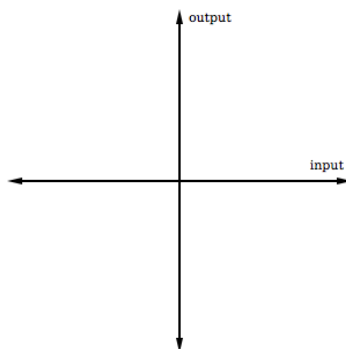
- Graph shape: _____
- Domain of $y = a^x$: **x is any real number**
- Range of $y = a^x$: **$y > 0$**

Examples of Exponential Functions

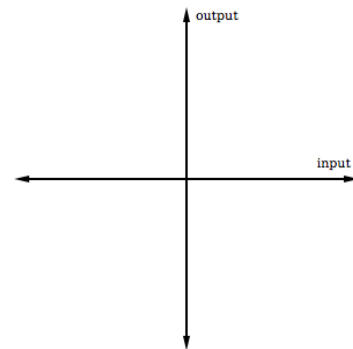
1. $y = \left(\frac{1}{4}\right)^x$



2. _____



3. _____



Notes about Exponential Family

- Possible number of x -intercepts: 0 1 2 3
- Possible number of y -intercepts: 0 1 2



1.3: Properties of Exponents

Learning Objectives

Together with your team:

- Look for patterns and apply definitions of exponents to develop Properties of Exponents.
- Apply properties of exponents to simplify or evaluate exponential expressions.

In this lesson, we will develop properties of exponents that will help us, for example, when evaluating exponential functions, or rewriting expressions involving exponents.

1) What is the mathematical meaning of the expression, 5^4 ? Write a sentence.

2) Rewrite 5^4 as a product of factors.

In this expression, the number 5 is called the **base** and 4 is called the **exponent**.

3) Let's look for a pattern.

a) Rewrite each exponential expression given below as a product, then evaluate the expression.

$$5^4 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$5^3 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$5^2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$5^1 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

b) Using specific examples to look for patterns can help us understand general properties of exponents. What pattern do you notice in the results you found in a)?

c) Extend the pattern to find the next step.

$$5^0 = \underline{\hspace{2cm}}$$

d) Generalize what you discovered in c), for any *non-zero* base, a .

e) Notice in d), we specified that the base, a , cannot be equal to 0. To see why, fill in each blank in the two patterns below.

$$0^4 = \underline{\hspace{2cm}}$$

$$4^0 = \underline{\hspace{2cm}}$$

$$0^3 = \underline{\hspace{2cm}}$$

$$3^0 = \underline{\hspace{2cm}}$$

$$0^2 = \underline{\hspace{2cm}}$$

$$2^0 = \underline{\hspace{2cm}}$$

$$0^1 = \underline{\hspace{2cm}}$$

$$1^0 = \underline{\hspace{2cm}}$$

According to this pattern,
 0^0 should be ____.

According to this pattern,
 0^0 should be ____.

Because these two patterns are not consistent, we say that 0^0 is **undefined**. In other words, the general property you wrote in d) does not apply when the base is 0.

4) Next, let's use specific examples again to understand the **Product Property of Exponents**.

a) Complete the following statement about multiplying factors with the same base:

By definition, 3^4 means _____ and 3^2 means _____, so $3^4 \cdot 3^2$ means _____.

Write this expression with a single exponent:

b) Generalize the example from a) to complete the **Product Property of Exponents**.

Let a , m , and n be any real numbers. Then, $a^m \cdot a^n = \underline{\hspace{2cm}}$

c) Use the Product Property of Exponents from b) to find the following products. Write your answer with a single exponent.

$$4^3 \cdot 4^7 = \underline{\hspace{2cm}}$$

$$2^4 \cdot 8^3 = \underline{\hspace{2cm}}$$

d) Why can't we apply the Product Property directly when rewriting the product, $2^4 \cdot 8^3$, in c)?

5) Next, try this example to discover another property of exponents.

a) By definition, a^3 means _____, so $(5^4)^3$ means _____.

Use one of your previous exponent properties to write this expression with a single exponent:

b) Generalize the example from a) to complete the **Power of a Power Property**.

Let a , m , and n be any real numbers. Then, $(a^m)^n =$ _____

6) Consider again the base 5 exponential expressions:

$$5^4 = 625$$

$$5^3 = 125$$

$$5^2 = 25$$

$$5^1 = 5$$

$$5^0 = 1$$

a) Extend the pattern to find the next three steps.

$$\text{_____} = \text{_____}$$

$$\text{_____} = \text{_____}$$

$$\text{_____} = \text{_____}$$

b) Generalize what you discovered in a), for any *negative exponent*.

7) Use the properties of exponents to rewrite the expression with a positive exponent only.

$$(2^3)^{-4} = \text{_____}$$

- 8) Reasoning about algebraic properties through the use of specific examples can help you recall a forgotten rule, when as it often does, memory fails. Next, create specific examples that help you understand each given property of exponents.

Properties of Exponents Reference Guide

Name	Property	Example
Product	$a^m \cdot a^n = a^{m+n}$	
Quotient	$\frac{a^m}{a^n} = a^{m-n}$	
Power of a power	$(a^m)^n = a^{m \cdot n}$	
Power of a product	$(a \cdot b)^n = a^n \cdot b^n$	
Power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	
-1 as an exponent	$a^{-1} = \frac{1}{a}$	
Zero as an exponent	$a^0 = 1$ if $a \neq 0$	



1.4: Algebra Critique – Properties of Exponents

Learning Objectives

Together with your team:

- Critique the applicability of a mathematical approach or the validity of a mathematical conclusion.
- Apply properties of exponents to simplify expressions.

A group of Algebra students were working on an ALEKS prep assignment for next week. They are all working on different topics and want to make sure that their answers are valid before entering them into ALEKS. Your task is to decide whether each student's answer is valid or not. **If an answer is invalid, explain why and correct the student's work.**

1) The first student is working on the following ALEKS problem.

☰
● EXPONENTS, POLYNOMIALS, AND FACTORING

Introduction to the product rule with negative exponents

Simplify.

$$w^{-6} \cdot w^{-8}$$

Write your answer with a positive exponent only.

The student's work:

$$w^{-6} \cdot w^{-8} = w^{-6 \cdot -8} = w^{48}$$

The student wants to enter in ALEKS:

$$w^{48}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

2) The second student is working on the following ALEKS problem.

☰
● EXPONENTS, POLYNOMIALS, AND FACTORING

Rewriting an algebraic expression without a negative exponent

Rewrite the expression without using a negative exponent.

$$6v^{-2}$$

Simplify your answer as much as possible.

The student's work:

$$6v^{-2} = \frac{6}{v^2}$$

The student wants to enter in ALEKS:

$$\frac{6}{v^2}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

3) The third student is working on the following ALEKS problem.

☰

●
EXPONENTS, POLYNOMIALS, AND FACTORING

Product rule with positive exponents: Univariate

▼

Multiply.

$$6x(-5x^4)$$

Simplify your answer as much as possible.

The student's work:

$$6x \cdot -5x^4 = 6 \cdot -5 \cdot x \cdot x^4 = -30x^5$$

$$= \frac{x^5}{30}$$

The student wants to enter in ALEKS:

$$\frac{x^5}{30}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

4) The fourth student is working on the following ALEKS problem.

☰

●
EXPONENTS, POLYNOMIALS, AND FACTORING

Power rules with negative exponents

▼

Simplify.

$$(3w^4x^{-5})^3$$

Write your answer using only positive exponents.

The student's work:

$$(3w^4x^{-5})^3$$

$$= 3w^{12}x^{-15} = \frac{3w^{12}}{x^{15}}$$

The student wants to enter in ALEKS:

$$\frac{3w^{12}}{x^{15}}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.



1.5: Function Notation

Learning Objectives

Together with your team:

- Use a table or equation to evaluate a function for a given input, and express the value using function notation (for example, find $f(-8)$ for $f(x) = x^2$).
- Interpret function values in the context of a given situation.

- 1) Sara purchased a new smartphone in 2016. The value, v , of Sara's phone is a function of the number of years, t , after she purchased the phone.

Using **function notation**, we could represent this relationship symbolically as:

$$\text{value} = v(t)$$

This notation means the "value is a function of time."

Suppose the value of Sara's smartphone t years after she purchased it can be modeled by the linear function:

$$v(t) = 650 - 150t.$$

- a) What is the input variable? Describe what it represents.

- b) What is the output variable? Describe what it represents.



Summary: Defining a Variable

- c) Make a table for the function showing several input/output pairs.

$$v(t) = 650 - 150t.$$

- d) According to the model, what is the value of Sara's smartphone after 3 years? (Write your answer as a complete sentence, including units.)

The function notation for this is: _____

- e) What does the function notation $v(t) = 0$ represent, in the context of the smartphone situation? Write a complete sentence.

- f) Solve for t :

$$v(t) = 0$$

- g) Explain in words, including units, what your answer to f) means in the context of the given situation. Write a complete sentence.

2) Let $g(x) = x^2$.

a) Evaluate $g(-4) = \underline{\hspace{2cm}}$

b) Write a sentence explaining what it means to “evaluate $g(-4)$.”

3) Let $h(x) = 5x - 12$.

a) Evaluate $h(13)$.

b) Solve for x : $h(x) = 13$

c) Write a sentence explaining what it means to “solve the equation $h(x) = 13$ for x .”

4) Let f be a function. Explain the difference between:

- “Evaluate $f(9)$ ” and
- “Solve $f(x) = 9$ for x .”

Summary: Function Notation



1.6: Translating Between Equations and Words

Learning Objectives

Together with your team:

- Translate between a symbolic representation and a verbal description of a function.

Desmos Class Code: _____

Recall...

- ✓ **A symbolic representation of a function** is an *equation* that shows how to determine the output from a given input, such as:

$$f(x) = \text{an expression in } x$$

- ✓ **A verbal description of a function** explains in *words* how to determine the output from a given input, such as:

“To find the value of the output, ...”

- 1) Record your answers to the Desmos matching game on Screen 4 here. The last two rows on the next page are for recording your answers to Screens 5 and 6.

Symbolic Representation of Function (Equation)	Verbal Description (Words)
	A To find the value of the output, multiply the input by two, then add six to the product.
	B To find the value of the output, add six to the input, then multiply the sum by two.
	C To find the value of the output, add six to the input, then square the sum.
	D To find the value of the output, square the input, then add six to the result.
	E To find the value of the output, add six to the input, then divide the sum by two.
	F To find the value of the output, multiply the input by six, then square the result.
	G To find the value of the output, divide the input by two, then add six to the quotient.
	H To find the value of the output, square the input, then multiply the result by six.
	I To find the value of the output, square the input, then add six squared to the result.

Symbolic Representation of Function (Equation)	Verbal Description (Words)
$f(x) = \sqrt{x + 25}$	
$h(t) = t^2 + 12t + 36$	

2) Now, challenge a classmate.

- a) First, partner up. Each partner creates a function rule, with real number inputs and outputs, and writes down a verbal description.

- b) Now, swap descriptions with your partner and try to represent the function they described using an equation.

Chapter Learning Objectives

1. Given the *graph* of a relation:
 - Identify the x -intercept(s) and the y -intercept, if any.
 - Evaluate the relation for a given input.
 - Solve for all inputs that have a given output.
 - Interpret inputs and outputs in the context of a situation.
 - Determine the relation's domain and range, and represent these on a number line and in interval notation.
 - Decide whether the relation is a function.
2. Interpret the slope of a given linear graph as a rate of change, and in the context of a situation.
3. Explain why a graph of a function is linear or nonlinear.
4. Model data using a graph.
5. Create a possible context (narrative) for a given graph.

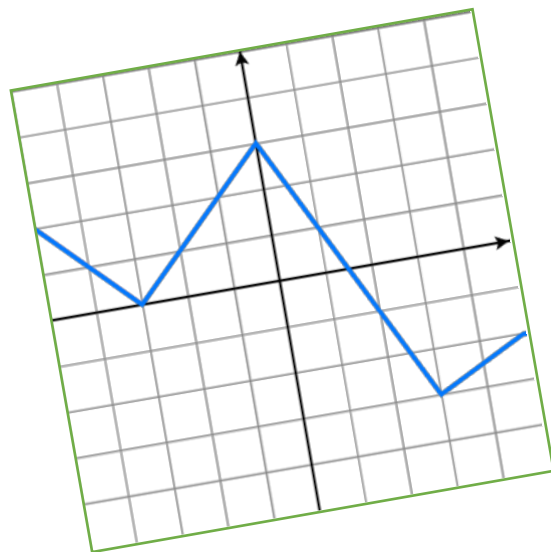
Chapter 2

What Can We Learn from a Graph?

Chapter Overview

Recall from Chapter 1, a graphical representation of a relation shows its ordered pairs plotted in the (x, y) plane. As we will see here in Chapter 2, graphs are useful tools that can help us answer, at a glance, many important questions about a given relation, such as:

- Is the relation a function?
- What are the domain and range of the relation?
- What are the x - and y -intercepts, if any?
- What is the value of the function for a certain input?
- For which inputs does the function have a certain value?
- How do the variables change relative to one another?
- How can we interpret the graph in context?



Chapter 2 Contents

Chapter Learning Objectives	25
Chapter Overview	25
2.1: Warm-Up.....	27
2.1: Inputs & Outputs from a Graph	29
2.2: Warm-Up.....	37
2.2: How Functions Change.....	38
2.3: Modeling Data with Graphs.....	43



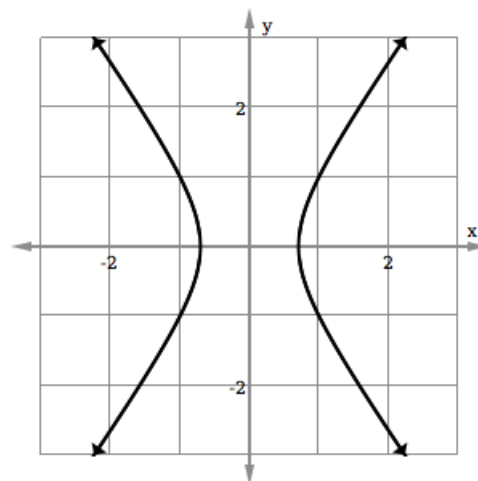
2.1: Warm-Up

An **interval** of real numbers is a continuous set on the number line. We commonly represent an interval using inequality notation, interval notation, or a verbal description.

- 1) For each of the sets given in the table below, sketch the set on the number line and fill in the missing representations.

Number line	Inequality notation	Interval notation	Verbal description
	$x < 2$		
		$[-5, 0)$	
			All real numbers strictly less than 6 AND strictly greater than -3
		$(-\infty, -4] \cup [4, \infty)$	
			All real numbers

2) The relation graphed below does NOT define y as a *function* of x . Justify why this is the case.





2.1: Inputs & Outputs from a Graph

Learning Objectives

Together with your team, given the *graph* of a relation:

- Identify the x -intercept(s) and the y -intercept, if any.
- Evaluate the relation for a given input.
- Solve for all inputs that have a given output.
- Interpret inputs and outputs in the context of a situation.
- Determine the relation's domain and range, and represent these in interval notation.
- Decide whether the relation is a function.

- 1) The graph of $y = v(t)$ models the value, v , in dollars of Sara's smartphone as a function of the number of years, t , after she purchased the phone.



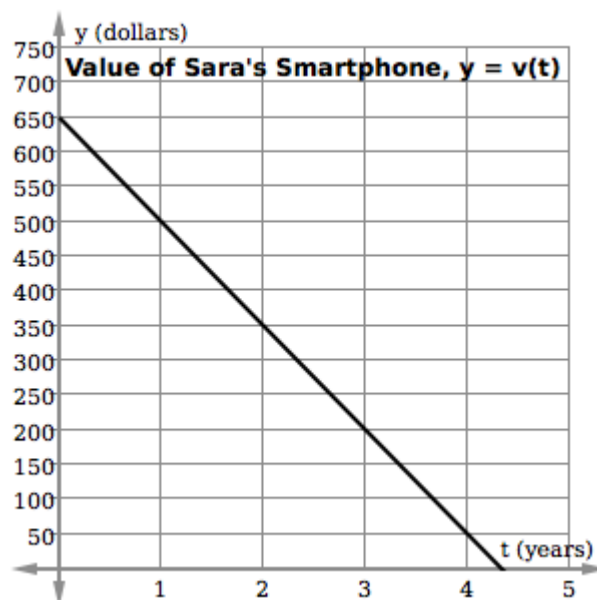
- a) Using the *graph*, identify the vertical intercept of the graph (y -intercept). Write the coordinates.

y -intercept: _____

The *function notation* used to represent this input/output pair is:

$v(\text{_____}) = \text{_____}$

- b) Explain the meaning of the y -intercept in the context of the given situation. Write a complete sentence, including units.



- c) Estimate the coordinates of the horizontal intercept of the graph (the t -intercept): _____

The *function notation* to represent this input/output pair is:

$v(\text{_____}) = \text{_____}$

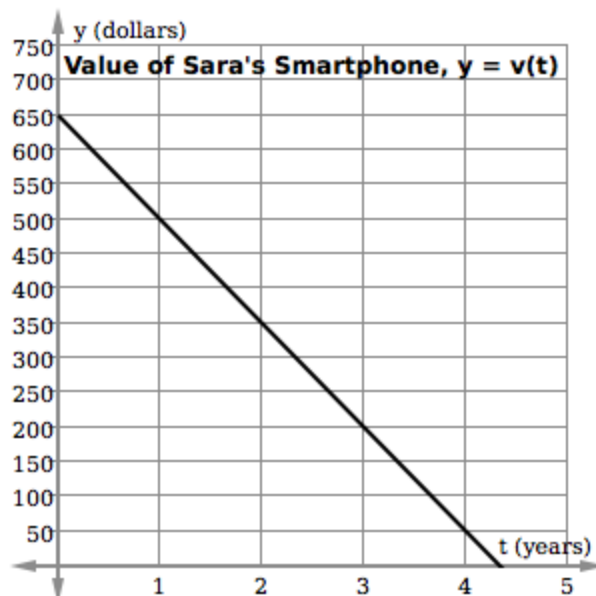
- d) Explain the meaning of the t -intercept in the context of the given situation. Write a complete sentence, including units.

- e) Show how to use the graph of v to solve for t when $v(t) = 200$.

$t =$ _____

- f) Explain in words, including units, what your answer to e) means in the context of the given situation. Write a complete sentence.

- g) The graph of $y = v(t)$ is drawn without arrows on each end. Why not?



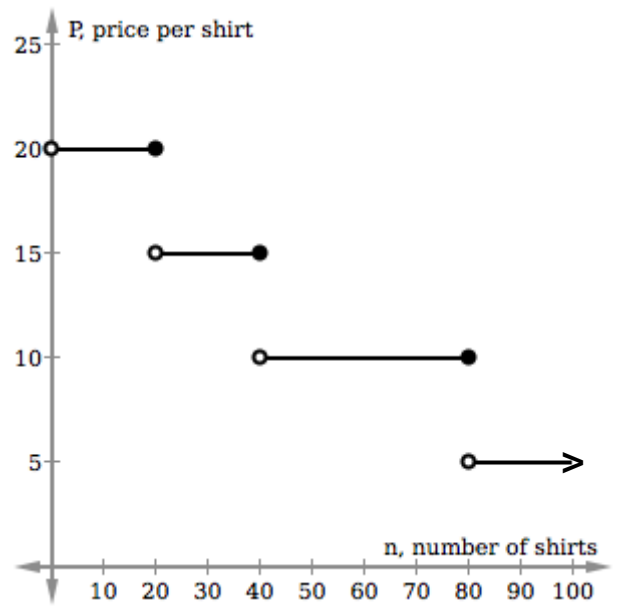
- h) What do you think is a reasonable domain and range for the function v in the context of the given situation? Write your answer in *interval* notation.

Summary: How to Determine these Features Given a Graph

Domain & Range	x -intercept(s) and y -intercept

2) The graph of $y = P(n)$ shows the price per shirt, P , as a function of the number of shirts, n , purchased from Cool Shirts T-Shirt Company.

a) Evaluate $P(60)$ and write a sentence interpreting your answer in the context of this situation.



b) Evaluate $P(20)=$ _____

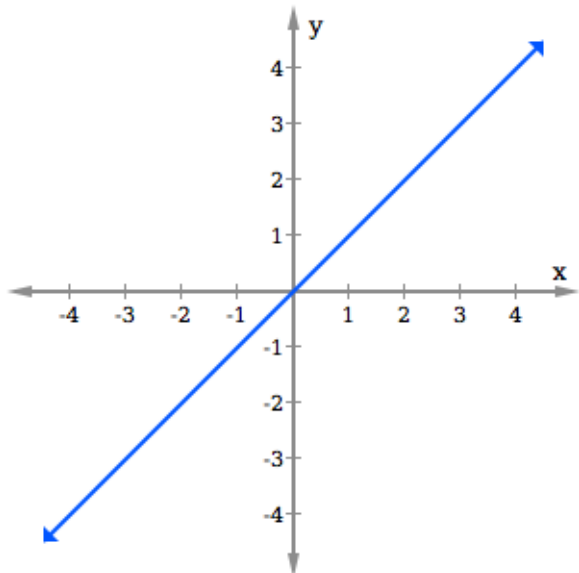
c) What is a reasonable domain of P ?

d) What is a reasonable range of P ?

For **3) – 7)**, consider the graphs of our five parent functions, $y = f(x)$. Use the graph to do the following:

- Identify the x -intercept(s) and the y -intercept, if any.
- Determine the relation's domain and range, and represent these in interval notation.
- Use the graph to evaluate the function at the specified values, if possible. If not possible, explain why.
- Use the graph to solve the given equations for x , if possible. If not possible, explain why.

3)



a) x -intercept(s): _____

y -intercept: _____

b) Domain: _____

Range: _____

c) $f(2) =$ _____ $f(-2) =$ _____

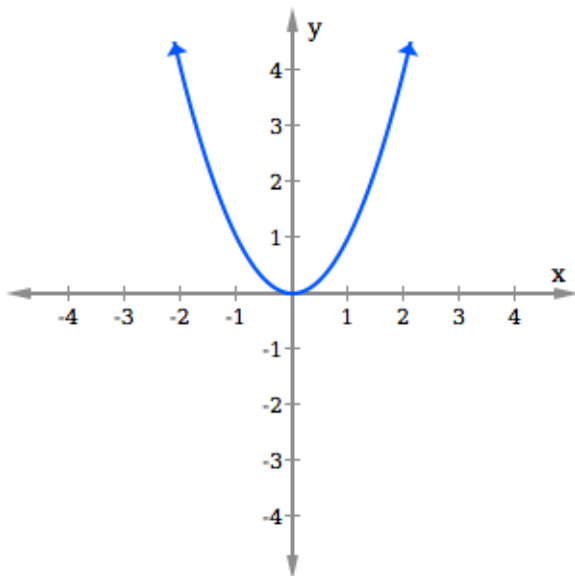
d) Solve for x : $f(x) = 1$

$x =$ _____

Solve for x : $f(x) = -1$

$x =$ _____

4)



a) x -intercept(s): _____

y -intercept: _____

b) Domain: _____

Range: _____

c) $f(2) =$ _____ $f(-2) =$ _____

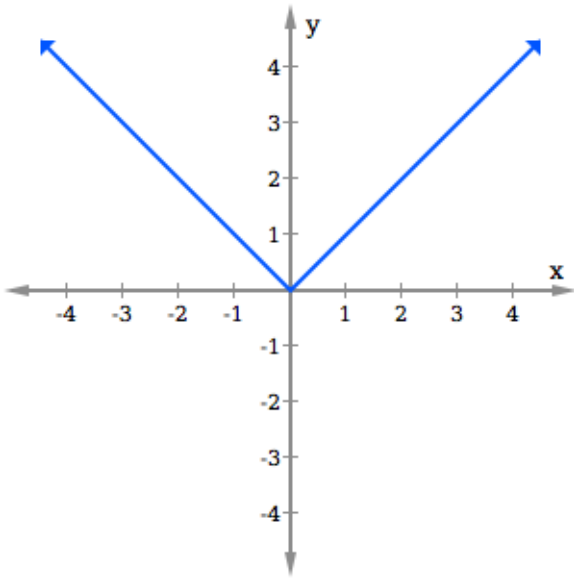
d) Solve for x : $f(x) = 1$

$x =$ _____

Solve for x : $f(x) = -1$

$x =$ _____

5)

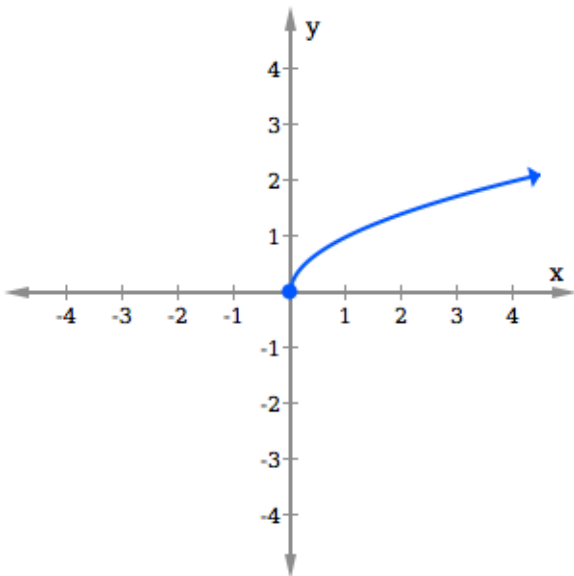
a) x -intercept(s): _____ y -intercept: _____

b) Domain: _____

Range: _____

c) $f(4) =$ _____ $f(-4) =$ _____d) Solve for x : $f(x) = 3$ $x =$ _____Solve for x : $f(x) = -3$ $x =$ _____

6)

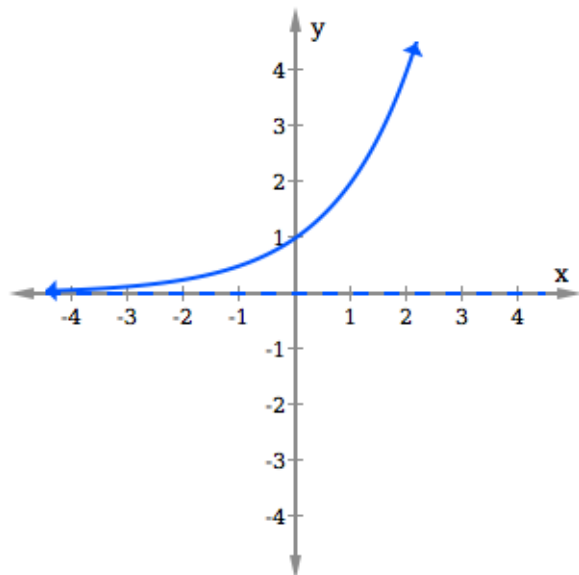
a) x -intercept(s): _____ y -intercept: _____

b) Domain: _____

Range: _____

c) $f(1) =$ _____ $f(-1) =$ _____d) Solve for x : $f(x) = 2$ $x =$ _____Solve for x : $f(x) = -2$ $x =$ _____

7)

a) x -intercept(s): _____ y -intercept: _____

b) Domain: _____

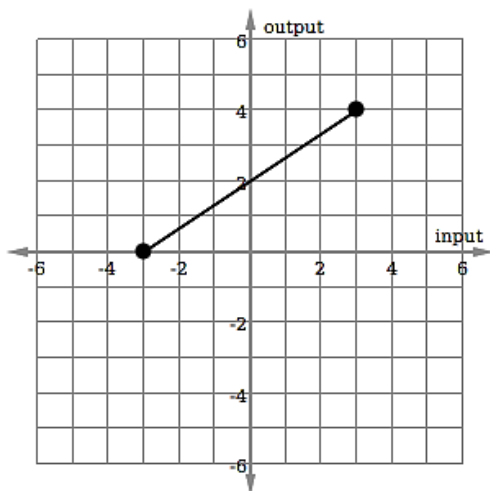
Range: _____

c) $f(2) =$ _____ $f(-2) =$ _____d) Solve for x : $f(x) = 4$ $x =$ _____Solve for x : $f(x) = -4$ $x =$ _____

For **8) – 11)**, use the graph of the relation to do the following:

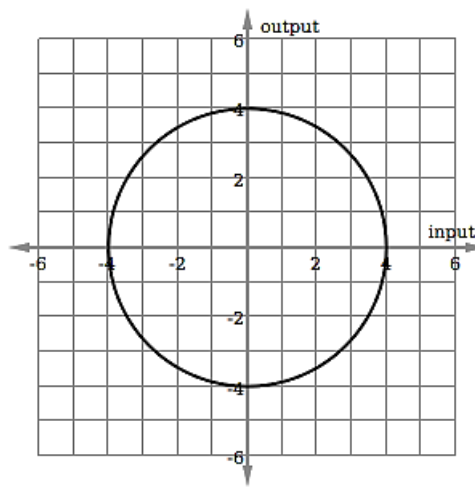
- Estimate the relation's domain and range, and represent these in interval notation.
- State whether the relation is a function.
- Estimate the x -intercept(s) and the y -intercept(s), if any.

8)



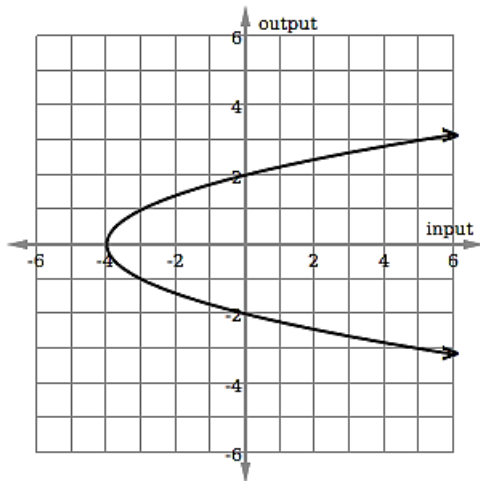
- Domain: _____ Range: _____
- Function? YES NO
- x -intercept(s): _____
 y -intercept(s): _____

9)



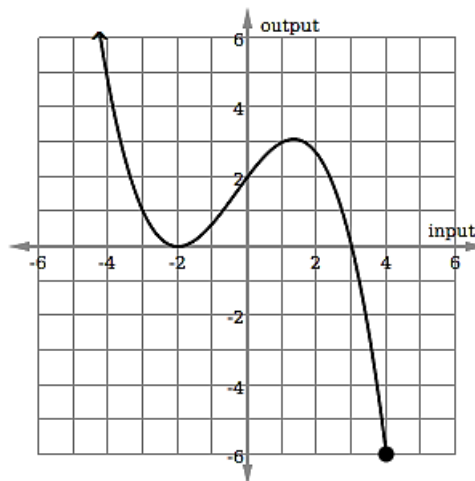
- Domain: _____ Range: _____
- Function? YES NO
- x -intercept(s): _____
 y -intercept(s): _____

10)



- Domain: _____ Range: _____
- Function? YES NO
- x -intercept(s): _____
 y -intercept(s): _____

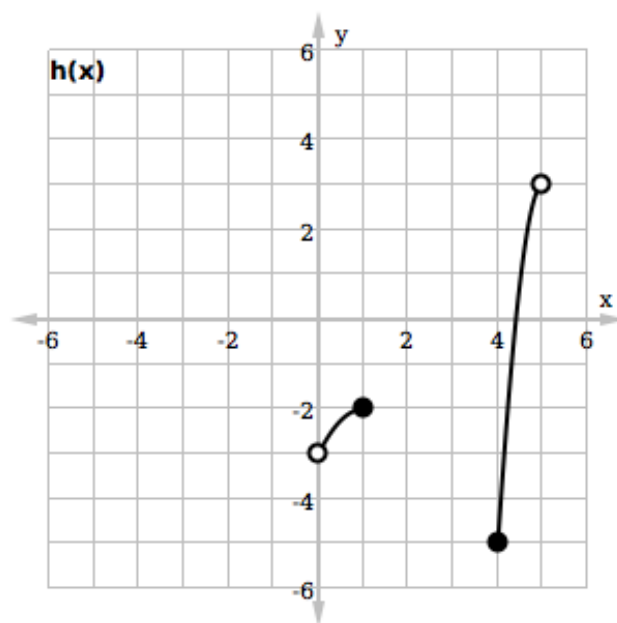
11)



- Domain: _____ Range: _____
- Function? YES NO
- x -intercept(s): _____
 y -intercept(s): _____

12) Use the graph of $y = h(x)$ to answer the following questions.

- a) What is the domain of h ? Write your answer in interval notation.
- b) What is the range of h ? Write your answer in interval notation.
- c) Estimate the coordinates of any x -intercept(s).
- d) What are the coordinates of the y -intercept, if any?
- e) What is the value of $h(0)$?
- f) Estimate the solution to the equation $h(x) = 0$?
- g) What is the value of $h(4)$?





2.2: Warm-Up

1) Consider a function that has the following properties:

- When the input is 0, the output is 4.
- Each time the input increases by 3, the output increases by 7.

a) Sketch a graph of a function that has the properties described.

b) Explain how the properties in the two bulleted items above are represented in the graph you sketched in a).



2.2: How Functions Change

Learning Objectives

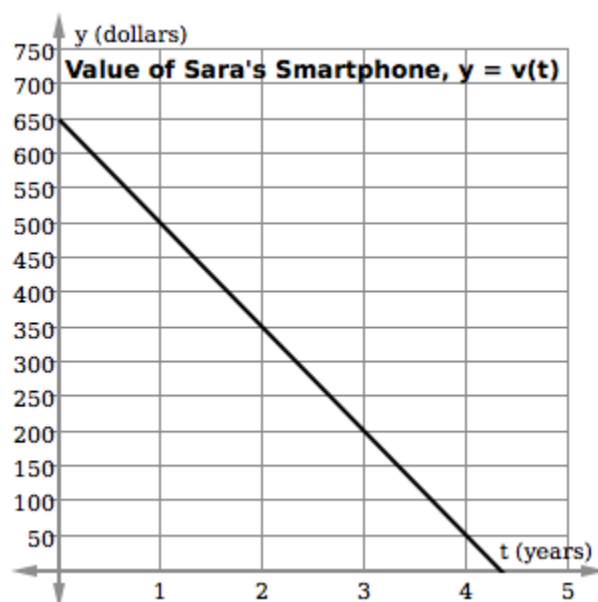
Together with your team:

- Interpret the slope of a linear graph as a rate of change.
- Describe the rate of change of the graph of a linear function, and interpret in the context of a situation.
- Explain why a function is linear or non-linear.

- 1) Consider again the graph of $y = v(t)$ that models the value, v , in dollars of Sara's smartphone as a function of the number of years, t , after she purchased the phone.



- a) Describe how the value of Sara's smartphone changes over time. Include units in your answer.

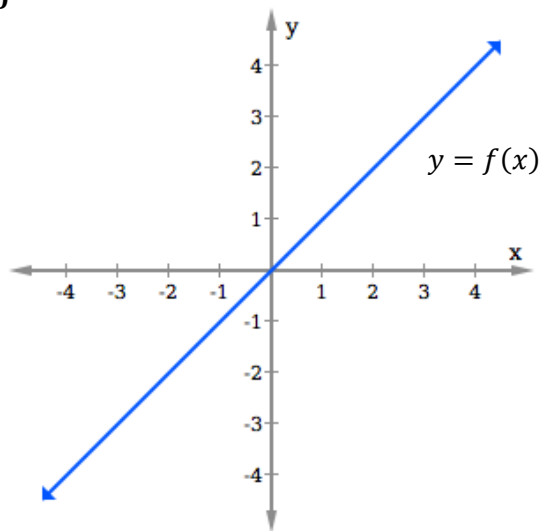


- b) Show how the change you described in a) is represented in the graph of $y = v(t)$.

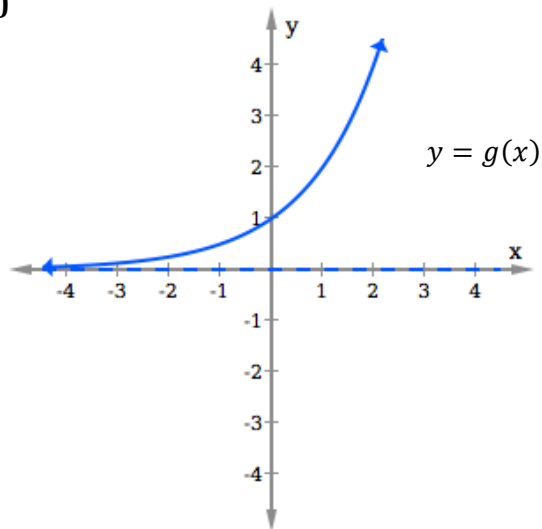
Shown below in **2)** and **3)** are graphs of our parent functions, $f(x) = x$ and $g(x) = 2^x$.

For each function, write a sentence to describe how the y -value changes each time the x -value increases by one unit.

2)



3)

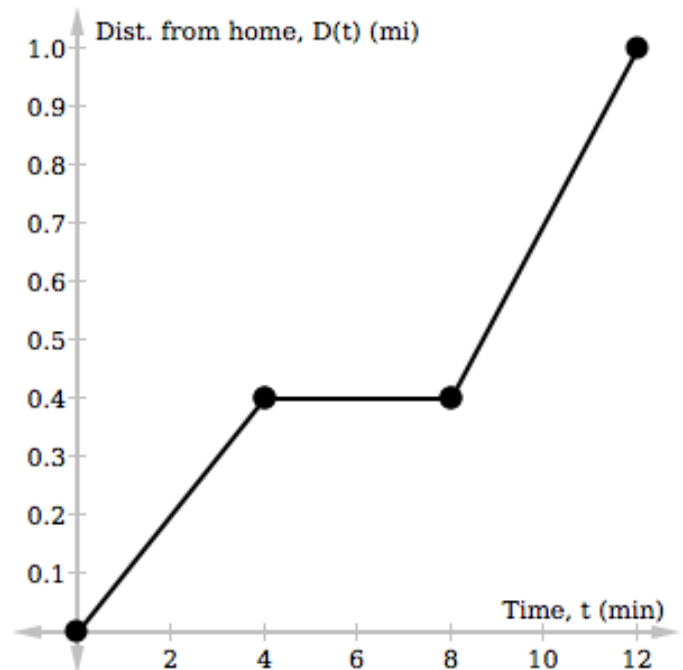


Summary: Rate of change

Linear change

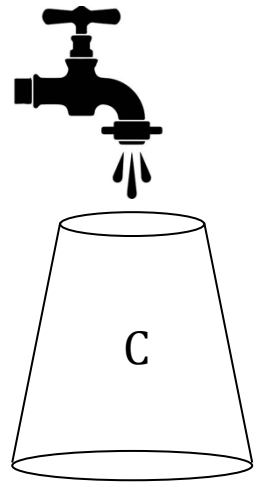
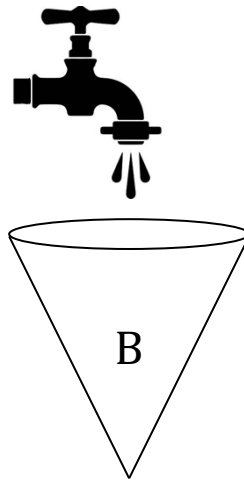
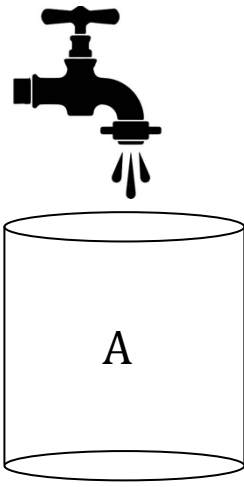
Exponential change

- 4) Sketch a graph of one of our parent functions with a rate of change that is neither linear nor exponential. Explain or show how the graph illustrates this.
- 5) The graph below models Katy's journey from her house to work each morning. She always stops at the coffee shop along the way. Her distance from home, $D(t)$, in miles, is represented as a function of the number of minutes, t , since she left home.
- a) What does the point $(0,0)$ represent in this situation?
- b) During which part of the trip is Katy traveling the fastest? What is her speed during this part of the trip? Explain.
- c) During which parts of the trip is Katy getting farther from home? Explain.
- d) What is Katy's speed during the time period from 4 to 8 minutes since she left home? Include units in your answer. Explain.



6) Suppose water is pouring at a constant rate into the three containers with the same height shown below.

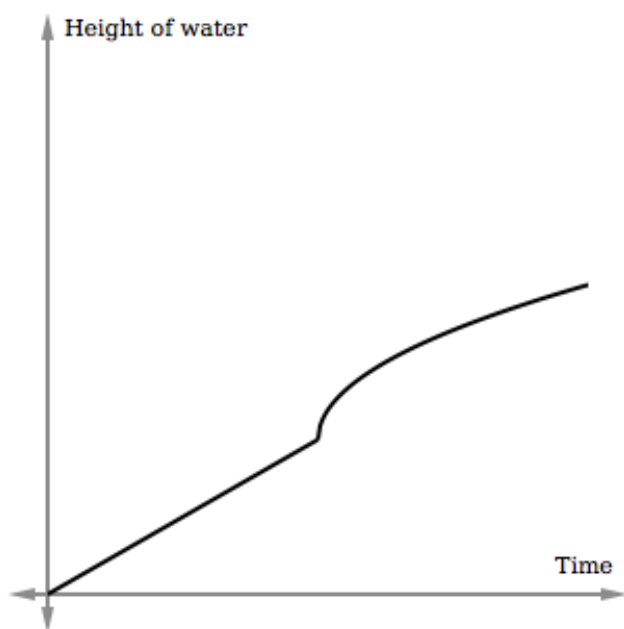
- a) First, think privately: If we were to use a graph to model the height of the water in the container, as a function of time, which of these container(s) do you think would have a *linear* graph?



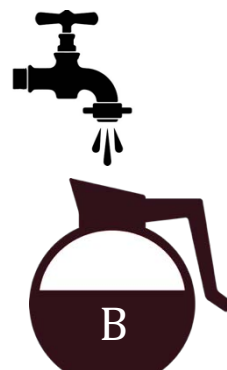
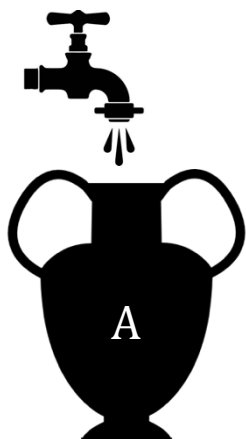
- b) For each container, sketch a graph of the height of water as a function of time. Then, write a brief explanation for each graph.

7) Consider again a situation where water is poured at a constant rate into a container.

- a) The graph below models the height of the water in a container, as a function of time. Draw a possible container for the graph.



- b) Choose one of the containers and sketch a graph of the height of the water in the container as a function of time. Then, show a classmate your graph and see if they can match your graph to the container you chose.





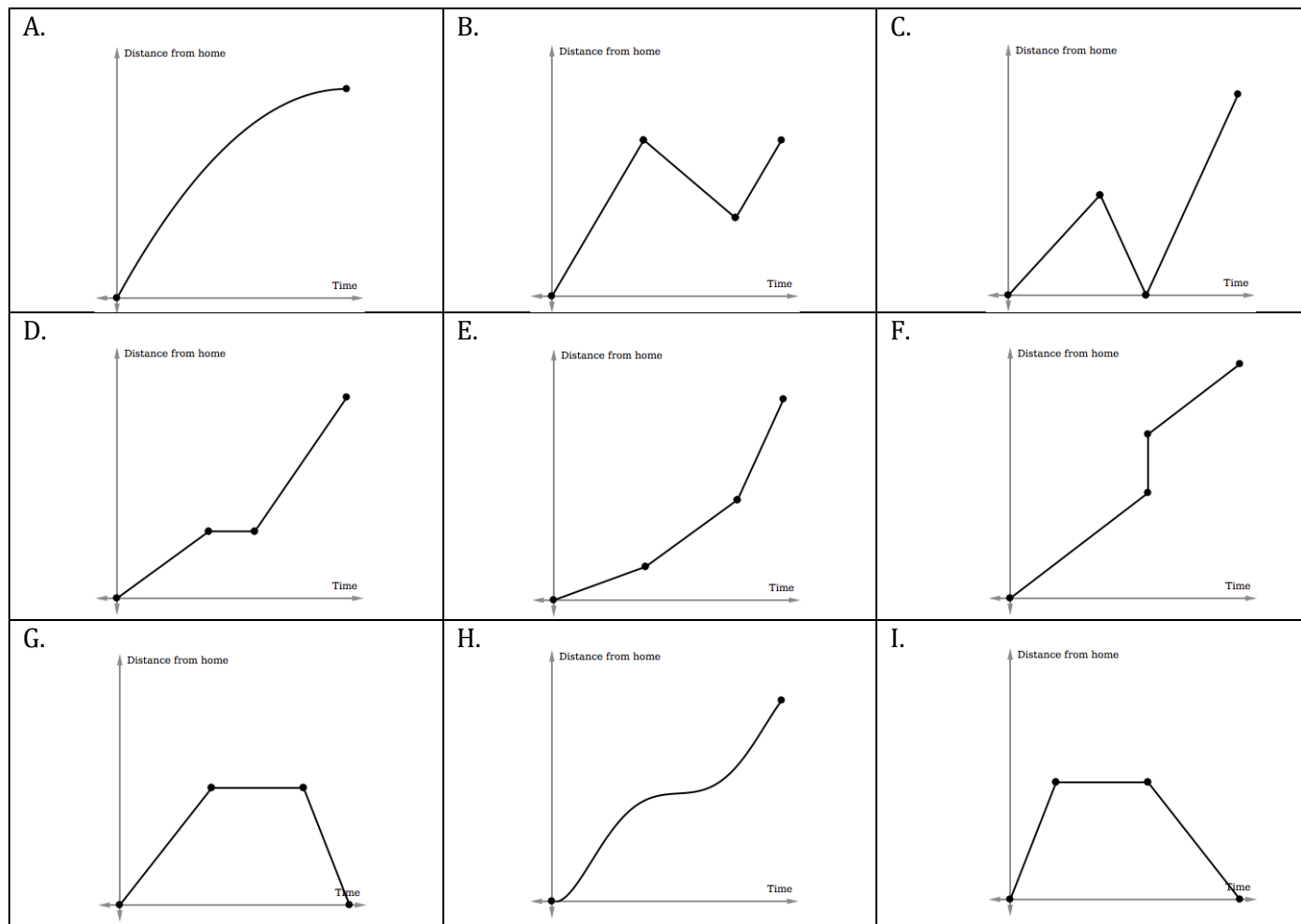
2.3: Modeling Data with Graphs

Learning Objectives

Together with your team:

- Match a given narrative to a graph that could be used to model the situation.
- Create a possible context (narrative) for the graph.

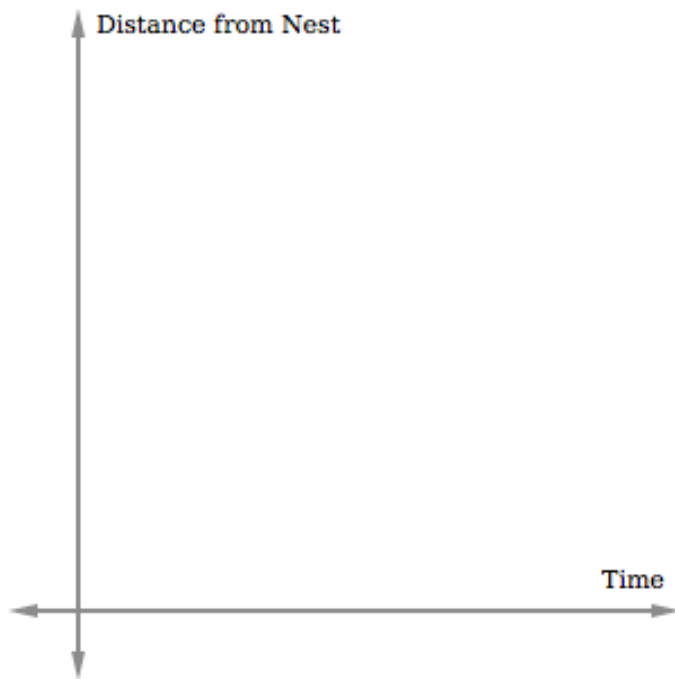
Find the graph that corresponds to each of the following narratives about what Lyn did after leaving her home. You will not use all the graphs. On each graph the x -axis represents time and the y -axis represents distance from home.



- _____ Lyn ran from her home to the bus stop and waited. After realizing she'd missed the bus, she walked home.
- _____ Lyn left her home for a run, but she wasn't feeling well and gradually came to a stop.
- _____ Opposite Lyn's home is a hill. She climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
- _____ Lyn went out for a walk with some friends. After realizing she'd left her wallet behind, she ran home to get it, and then ran to catch up with the others.
- _____ Lyn skateboarded from her house, gradually building up speed. She slowed down to avoid some rough ground, but then sped up again.
- _____ Lyn walked to the store at the end of her street, bought a newspaper, and then ran all the way back home.

7) Sketch a graph to model the following situation.

"An eagle leaves its nest to go hunting. It flies for several miles away from the nest before stopping to eat. After eating, it flies back to its nest to rest for a bit. After resting, the eagle flies away from its nest again, looking for more food."



8) Given below are three different narratives about what Lyn did after leaving her home.

Option 1:

Lyn took her dog for a walk to the park. She set off slowly and then increased her pace. At the park Lyn turned around and walked slowly back home.

Option 2:

Lyn rode her bike east from home up a steep hill. After a while the slope eased off. At the top, she raced down the other side.

Option 3:

Lyn went for a jog. At the end of her road she bumped into a friend and her pace slowed. When Lyn left her friend, she walked quickly back home.

a) Choose one of the narratives and sketch a graph to model the situation.

b) Next, show a classmate your graph and see if they can match your graph to the narrative you selected.

Chapter Learning Objectives

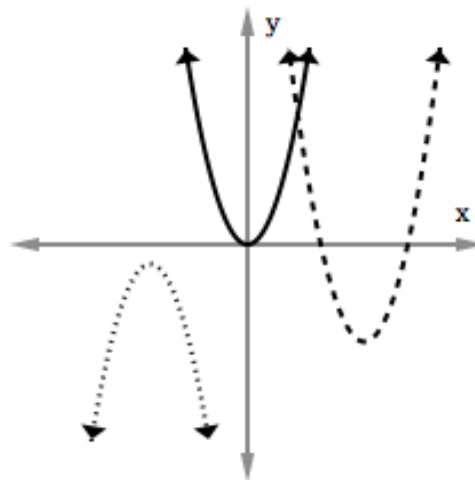
1. Describe how the parameters of linear, quadratic, and exponential functions influence the graphs.
2. Recognize, symbolically and graphically, the slope and y-intercept of a linear function.
3. Describe how the graph of a quadratic function and the symbolic representation in vertex form are related.
4. Given an equation in one of the following forms, sketch a graph of the function:
 - A linear function in slope-intercept form: $f(x) = ax + b$,
 - A quadratic function in vertex form: $f(x) = a(x - h)^2 + k$,
 - An exponential function in the form: $f(x) = a(b)^x$.
5. Given the equation of a quadratic function in vertex form, identify the coordinates of its minimum or maximum point and the equation of its axis of symmetry.
6. Given the equation of an exponential function in the form, $f(x) = a(b)^x$, sketch a graph that includes its y-intercept, and determine whether the output values increase or decrease as the inputs increase.

Chapter 3

How is a Parent Function Related to Other Members of its Family?

Chapter Overview

In this chapter, we will explore how the numerical values in the equation of a function, known as parameters, characterize the members of the linear, quadratic, and exponential function families. Throughout your explorations, be sure to look for connections between the symbolic forms and graphical forms of functions.



Chapter 3 Contents

Chapter Learning Objectives	45
Chapter Overview	45
3.1: Warm-Up.....	47
3.1: Influence of Parameters in Quadratic Functions.....	51
3.2: Influence of Parameters in Exponential Functions	57
3.3: Warm-Up.....	61
3.3: Algebra Critique–Influence of Parameters in Other Function Families	62



3.1: Warm-Up

In this Warm-up for Lesson 3.1, consider the Linear Family of functions. All the members of the Linear Family may be represented symbolically in the form:

$$f(x) = ax + b.$$

- 1) In the equation of a linear function, $f(x) = ax + b$, _____ is the input variable and _____ is the output variable.

In your own words, write what you think the word “variable” means in mathematics.

- 2) In the equation of a linear function, $f(x) = ax + b$, we call a and b “**parameters**.”

- a) For example, for the linear function, $g(x) = 2x + 5$, the parameters are $a =$ _____ and $b =$ _____.

In your own words, write what you think is meant by the word, “parameter” in mathematics.

- b) For the *parent* function of the Linear Family, $f(x) =$ _____, the parameters are $a =$ _____ and $b =$ _____.

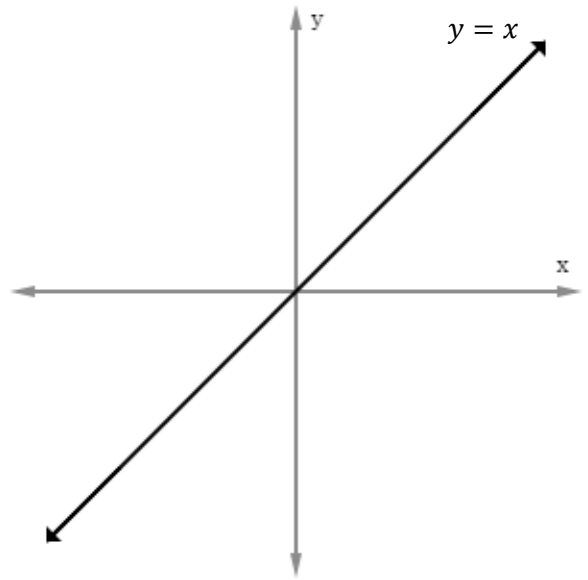
- 3) Next, you will use sliders in Desmos to see how different values of the parameters, a and b , influence the graphs of the members of the linear function family.

Open a Desmos graph using the link below.

<https://www.desmos.com/calculator/w15q5hlgju>

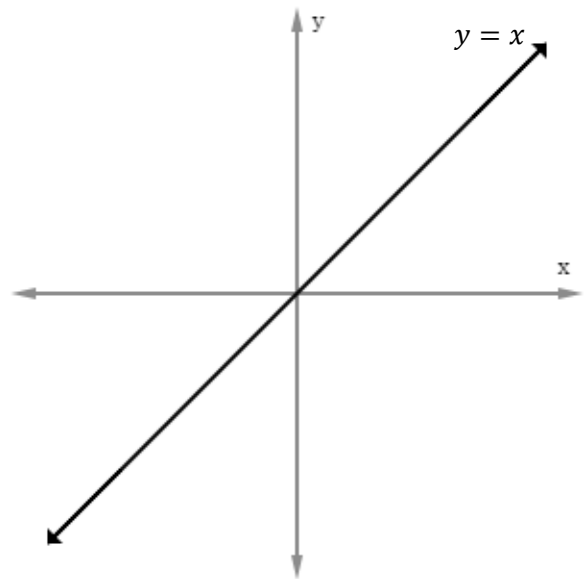
- a) Starting with the linear *parent* function, use the slider to change the value of the parameter a . How does changing the parameter a in the equation of a linear function influence its graph?

- b) Sketch two example graphs to support your answer to a). Also, include their equations.



- c) Starting again with the linear parent function, use the slider to change the value of the parameter b . How does changing the parameter b in the equation of a linear function influence its graph?

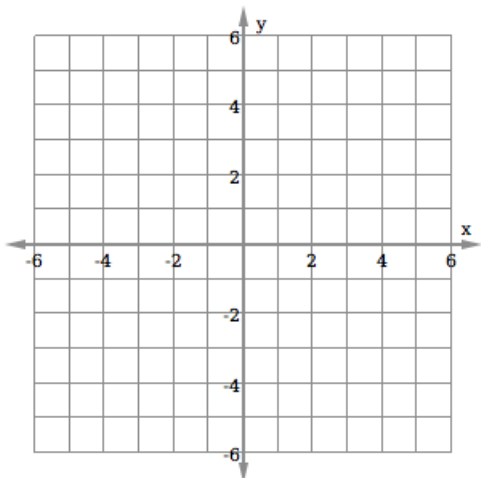
- d) Sketch two example graphs to support your answer to c). Also, include their equations.



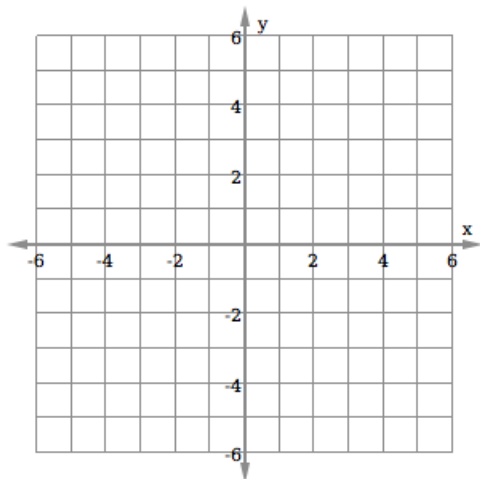
- 4) If you set the Desmos sliders to $a = -\frac{1}{2}$ and $b = -6$, how does the graph of $f(x) = ax + b$ compare to the graph of the parent function, $y = x$?

- 5) Using what you know about the role of the parameters a and b in a linear function, $f(x) = ax + b$, graph each function given below on the axes provided.

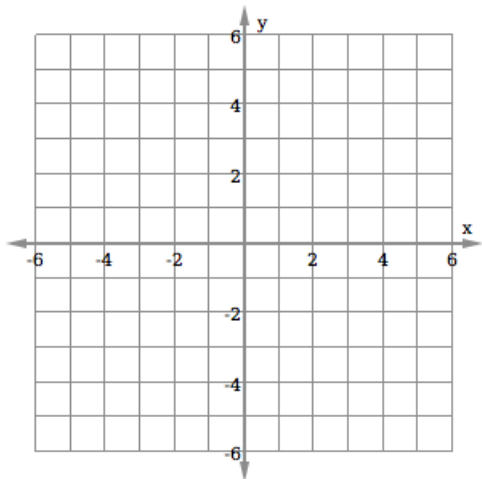
a) $h(x) = 3x$



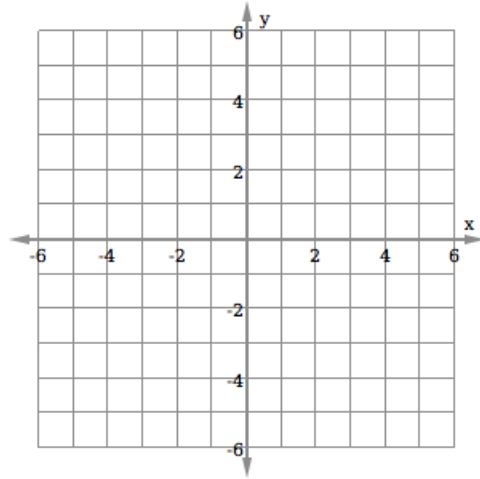
b) $g(x) = -\frac{1}{3}x - 2$



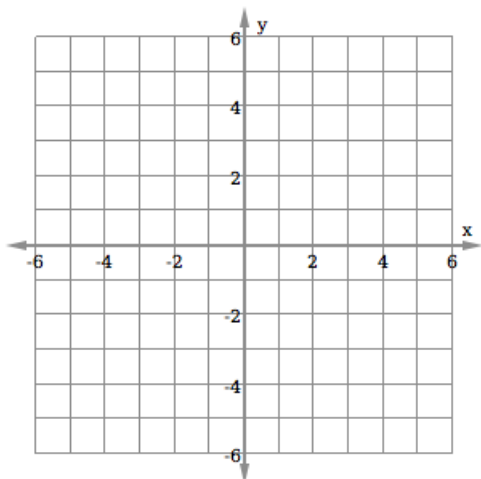
c) $f(x) = 2x + 5$



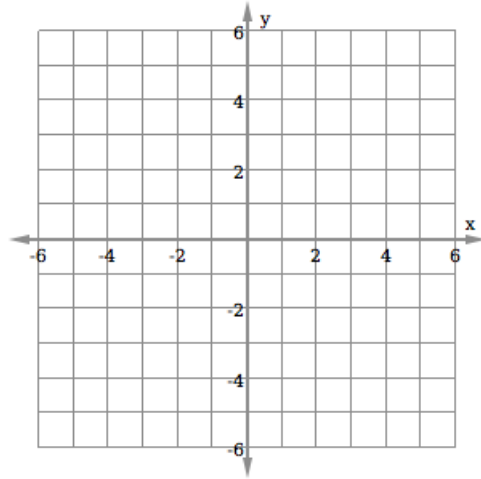
d) $j(x) = 2x - 6$



e) $k(x) = -x + 4$



f) $t(x) = x + 1$



Summary: Graphs of Linear Functions of the form, $f(x) = ax + b$ (or $f(x) = mx + b$)		
a	b	Parallel/Perpendicular lines



3.1: Influence of Parameters in Quadratic Functions

Learning Objectives

Together with your team:

- Describe how the parameters of a quadratic function, given in the form, $f(x) = a(x - h)^2 + k$, affect the graph.
- Describe how the graphical representation of a quadratic function and the symbolic representation in vertex form are related.
- Given the equation of a quadratic function in vertex form, identify the coordinates of its minimum or maximum point and the equation of its axis of symmetry.

Throughout this lesson, consider the Quadratic Family of functions. All the members of the Quadratic Family may be represented symbolically in the form:

$$f(x) = a(x - h)^2 + k.$$

- 1) If the values of the parameters are $a = 1$, $h = 0$ and $k = 0$, then $f(x) =$ _____.
- 2) As you did with linear functions in the 3.1 Warm-up, use Desmos sliders—one at a time—to see how different values of the parameters a , h , and k influence the graphs in the quadratic function family.

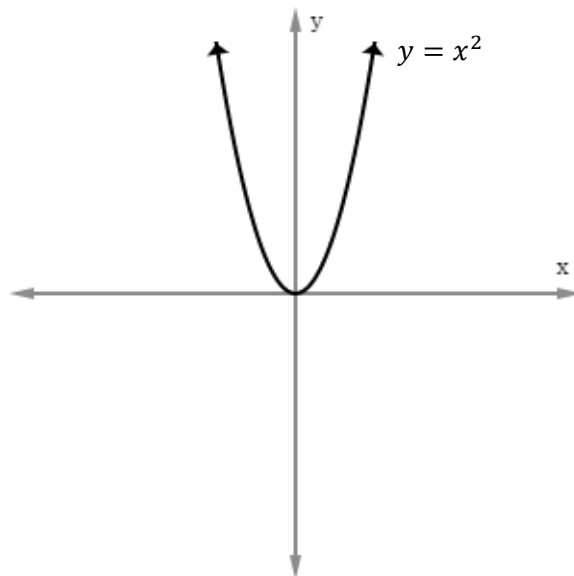
Open a Desmos graph using the link below.

<https://www.desmos.com/calculator/xgirbdgvpk>

- a) Starting with the quadratic *parent* function, use the slider to change the parameter a .

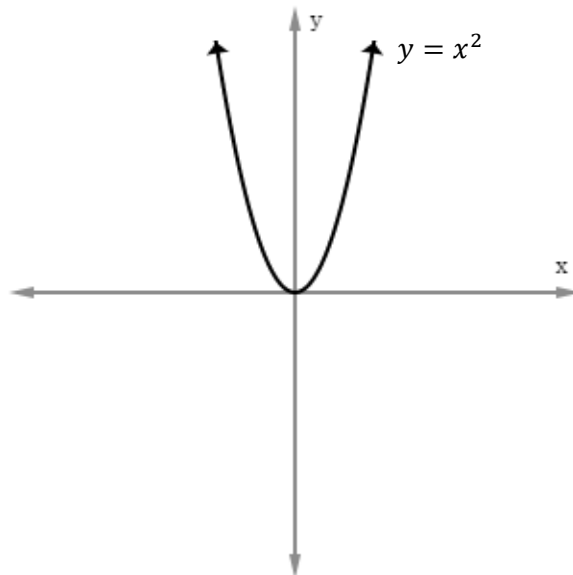
How does changing the parameter a in the equation of a quadratic function influence its graph?

- b) Sketch two example graphs to support your answer to a). Also, include their equations.



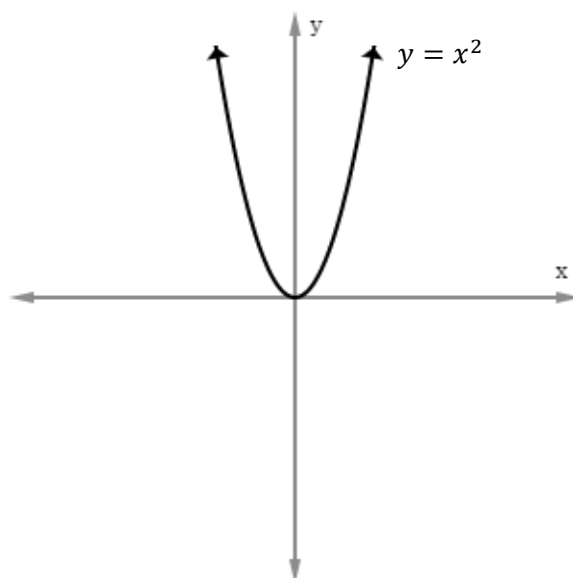
- c) Starting again with the quadratic *parent* function, use the slider to change the parameter h .
How does changing the parameter h in the equation of a quadratic function influence its graph?

- d) Sketch two example graphs to support your answer to c).
Also, include their equations.



- e) Finally, starting again with the quadratic *parent* function, use the slider to change the parameter k .
How does changing the parameter k in the equation of a quadratic function influence its graph?

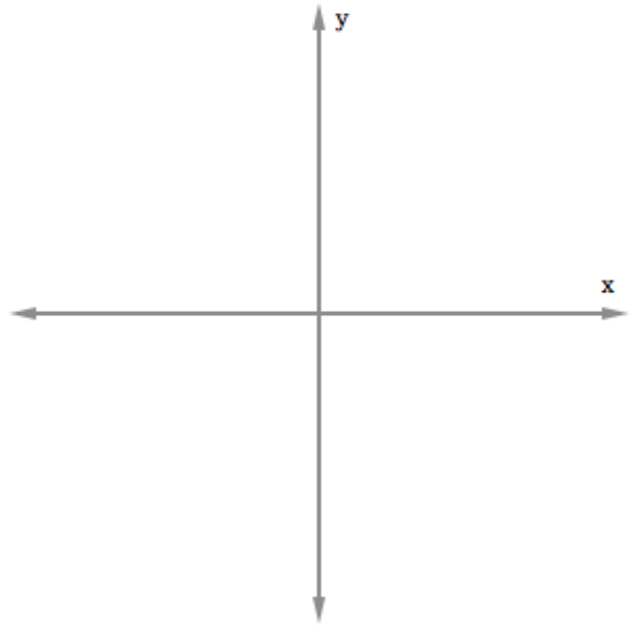
- f) Sketch two example graphs to support your answer to e).
Also, include their equations.



3) Let $p(x) = (x + 3)^2$.

a) How does the graph of p compare to the graph of $y = x^2$?

b) Sketch a graph that's good enough to illustrate your answer to a).



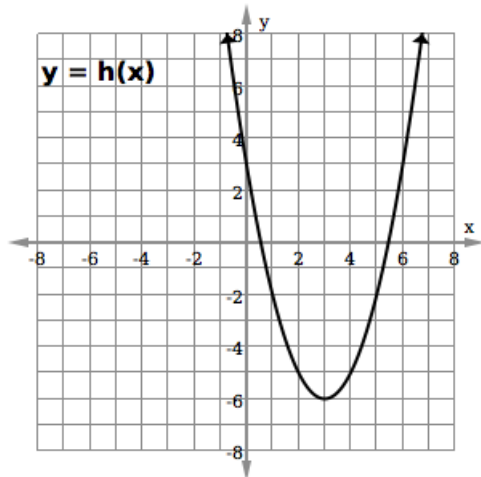
4) If you set the Desmos sliders to $a = -2$, $h = -1$ and $k = 4$ how does the graph of $f(x) = a(x - h)^2 + k$ compare to the graph of the parent function, $y = x^2$?

The graphs shown in 5) and 6) below represent functions from the Quadratic family, with the parameter $a = 1$ or $a = -1$.

- a) Label the (x, y) coordinates of the vertex of the parabola.
b) Write the equation of each function in the space provided.

5)

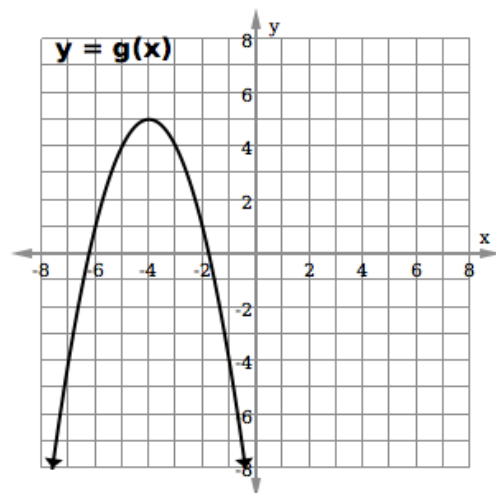
a)



b) $h(x) =$ _____

6)

a)



b) $g(x) =$ _____

7) Let $j(x) = -3(x + 7)^2 - 10$.

a) The parabola opens: Upward Downward

b) The parabola is Steeper or Less steep than $y = x^2$.

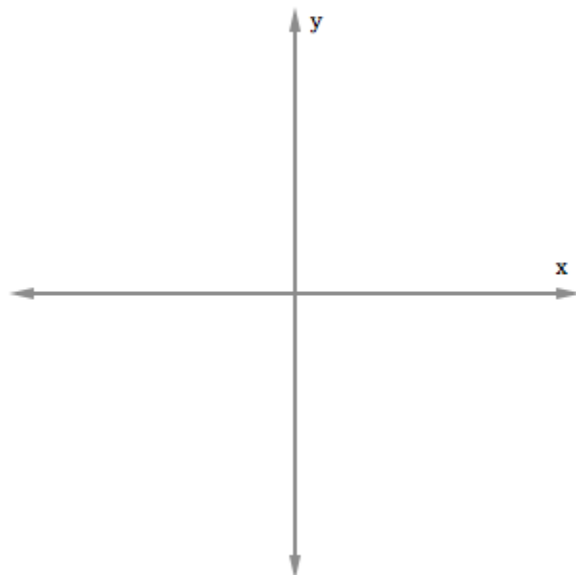
c) The (x, y) coordinates of the vertex are: _____

d) The vertex is the Maximum or Minimum point on the graph.

e) Explain your reasoning to d).

f) Write the equation for the axis of symmetry of the graph of j : _____

g) Sketch a graph of j that's good enough to illustrate your answers to a) – f).



Summary: Graphs of Quadratic Functions of the form, $f(x) = a(x - h)^2 + k$

a	h	k

- 8) The graph of a quadratic function, q , is a parabola that opens upward, is less steep than the graph of $y = x^2$, and has a vertex of $(12, -15)$.

a) What is the equation of the axis of symmetry of the parabola?

b) Write a *possible* equation for $q(x)$.

$$q(x) = \underline{\hspace{4cm}}$$

c) What additional information would we need to know if we wanted to write the equation of a specific parabola?

9) Give an example of a quadratic function, $f(x) = a(x - h)^2 + k$, with each of the following properties.

a) No x -intercepts

b) One x -intercept

c) Two x -intercepts



3.2: Influence of Parameters in Exponential Functions

Learning Objectives

Together with your team:

- Describe how the parameters of an exponential function, $f(x) = a(b)^x$, affect the graph.
- Given the equation of an exponential function in the form, $f(x) = a(b)^x$,
 - sketch a graph that includes its y-intercept, and
 - determine whether the output values increase or decrease as the inputs increase.

In this lesson, we will consider *some** members of the Exponential Family—those that can be represented symbolically in the form:

$$f(x) = a(b)^x, \text{ with } b > 0, b \neq 1, \text{ and } a > 0$$

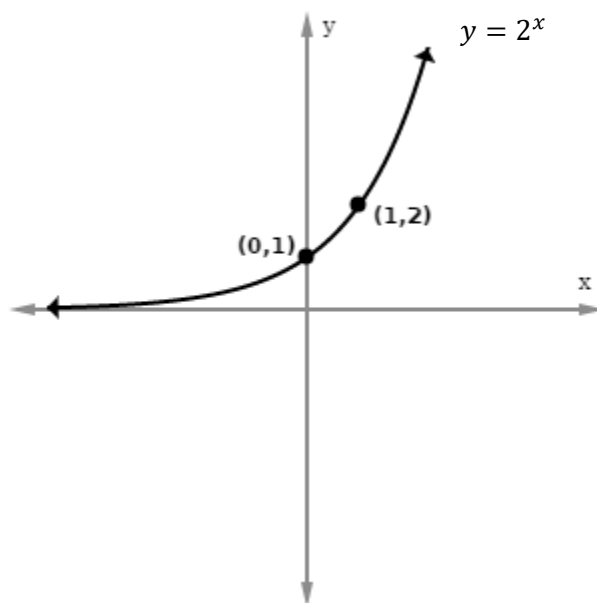
- 1) If the values of the parameters are $a = 1$ and $b = 2$ then $f(x) = \underline{\hspace{2cm}}$, our exponential parent function.
- 2) As you did in Lesson 3.1 use Desmos sliders—one at a time—to see how different values of the parameters, a and b , influence the graphs in the exponential family.

Open a Desmos graph using the link below.

<https://www.desmos.com/calculator/dnkutw5rc9>

- a) Starting with the *parent* function in **1)**, first use the slider to change the parameter b , the **base** of the exponential function.

How does changing the parameter b in the equation of an exponential function influence its graph?



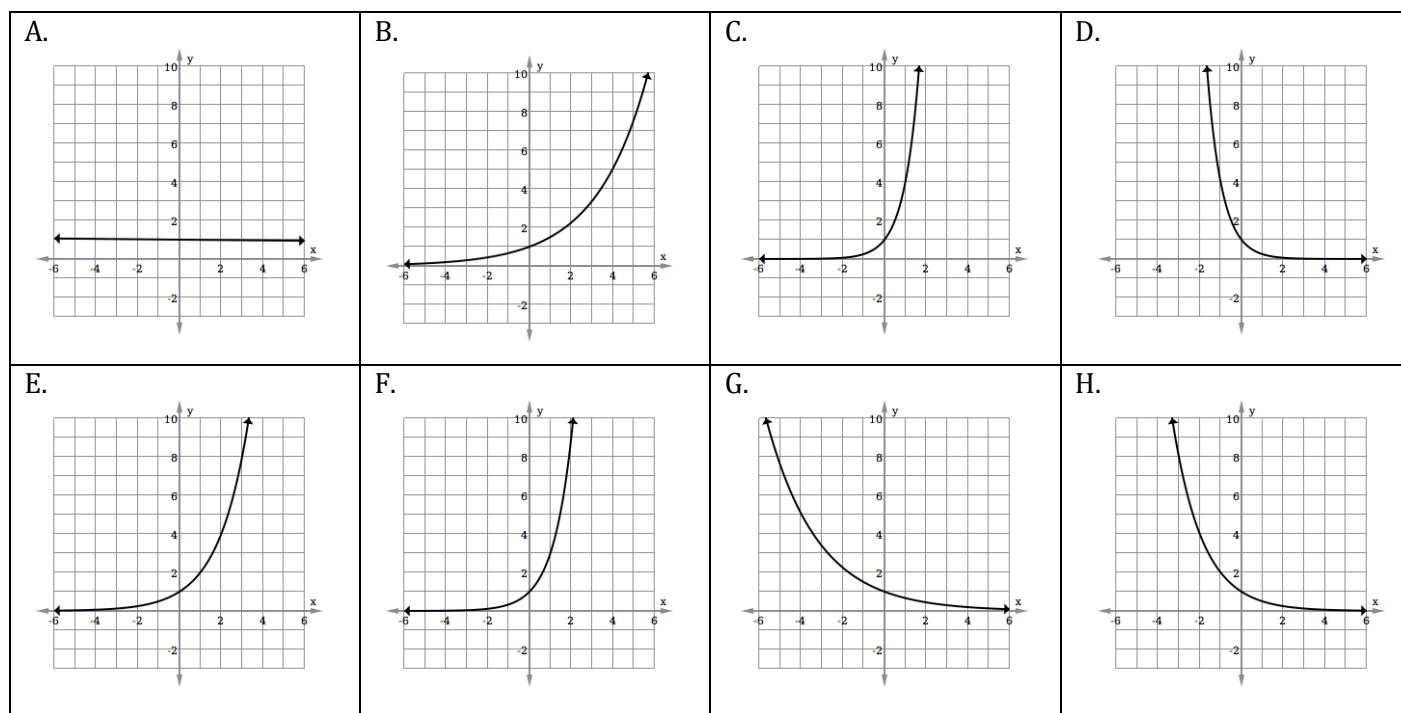
- b) Sketch two example graphs to support your answer to a). Also, include their equations.

*Note: the next course, *College Algebra*, deals with the other members of the Exponential Family.

c) Why do you think it makes sense to specify that the base of an exponential function is $b \neq 1$?

Here's more practice with graphs and equations of exponential functions that have different values of the base, b . Find the graphical representation for each of the symbolic representations of exponential functions given in **3) – 6)**. (Write the letter of the graph in the blank provided next to each equation.) You will not use all the graphs.

3) _____ $j(x) = 3^x$ **4)** _____ $s(x) = \left(\frac{1}{2}\right)^x$ **5)** _____ $m(x) = 0.99^x$ **6)** _____ $f(x) = \left(\frac{3}{2}\right)^x$

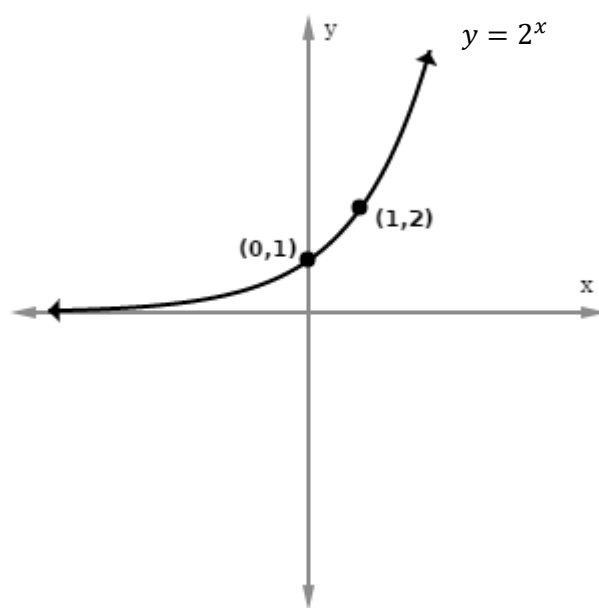


7) Look back at the graphs and equations in **3) – 6)**. What is the *same* about all the functions?

- 8)** Next, go back to Desmos and start with the exponential *parent* function. Use the slider to change the parameter a .

How does changing the parameter a in the equation of an exponential function influence its graph?

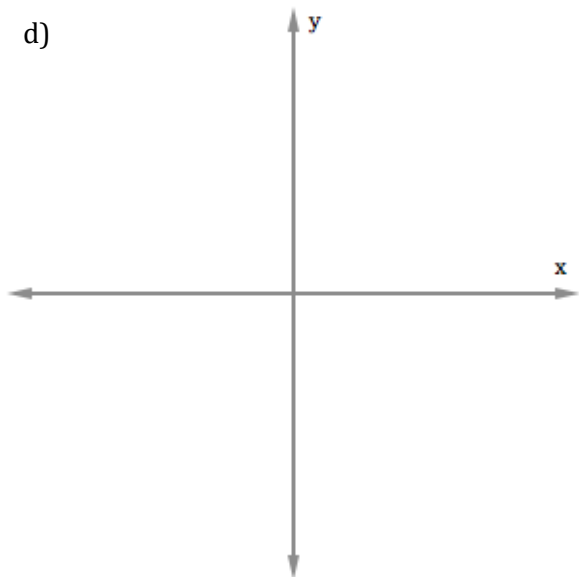
- 9)** Sketch two example graphs to support your answer to 8). Also, include their equations.



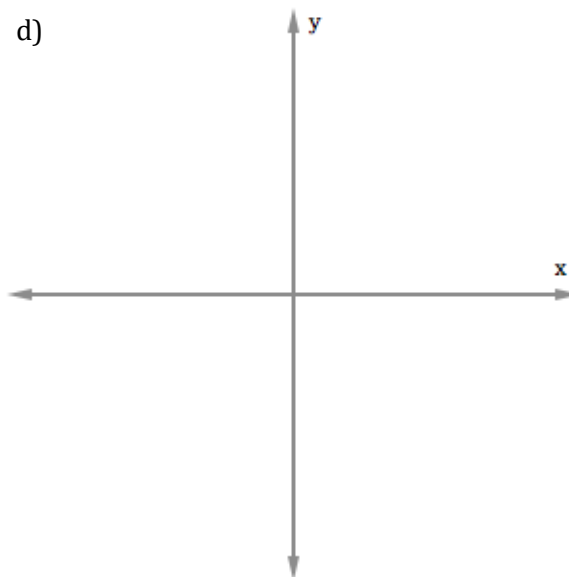
- 10)** If you set the Desmos sliders to $a = 5$ and $b = \frac{1}{2}$ how does the graph compare to the graph of the parent function, $y = 2^x$?

- Give the values of the parameters, a and b .
- State whether the function values increase or decrease as x increases.
- Give the (x, y) coordinates of the y -intercept of the graph.
- Sketch a graph that is good enough to illustrate your answers to a) – c).

d)



d)


$$b$$



3.3: Warm-Up

1) How does the graph of $g(x) = -x^2$ compare to the graph of $y = x^2$?

2) How does graph of $h(x) = -|x|$ compare to the graph of $y = |x|$?

3) How does the graph of $k(x) = -\sqrt{x}$ compare to the graph of $y = \sqrt{x}$?



3.3: Algebra Critique–Influence of Parameters in Other Function Families

Learning Objectives

Together with your team:

- Critique the applicability of a mathematical approach or the validity of a mathematical conclusion.
- Describe how the parameters of linear, quadratic, and exponential functions influence their graphs.

A group of Algebra students was working on some Wrap-up and ALEKS questions to study for their midterm exam. They are all working on different questions and want to make sure that their answers are valid before going to take the exam. Your task is to decide whether each student's answer is valid or not. **If an answer is invalid, explain why and correct the student's work.**

1) The first student is working on the following Wrap-up question and their answers are given below:

Determine the information about the given absolute value functions:

	$y = -4 x $	$y = \frac{1}{4} x $
a) Choose whether the graph opens upward or downward	<input type="radio"/> Upward <input checked="" type="radio"/> Downward	<input checked="" type="radio"/> Upward <input type="radio"/> Downward
b) Choose whether the graph is steeper or less steep than the graph of $y = x $	<input type="radio"/> Steeper <input checked="" type="radio"/> Less Steep	<input checked="" type="radio"/> Steeper <input type="radio"/> Less Steep

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's answer and explain.

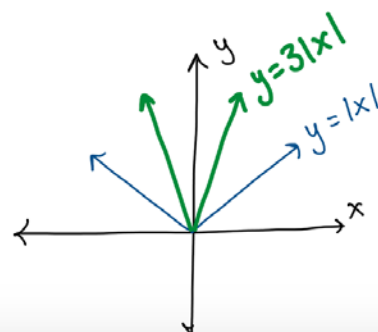
2) The second student is working on the following ALEKS problem:

Graph the function $g(x) = 3|x|$ along with a graph of the parent function $y = |x|$.

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work and explain.

The student wants to enter this graph in ALEKS:



3) The third student is working on the following ALEKS question:

GRAPHS AND FUNCTIONS
Translating the graph of an absolute value function: One step

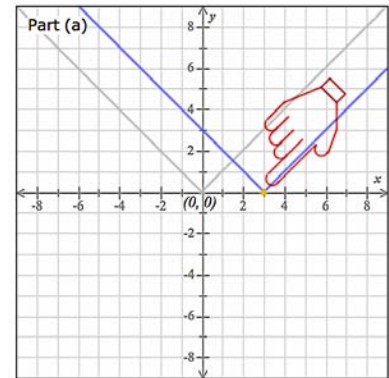
Translate each graph as specified below.

(a) The graph of $y = |x|$ is shown. Translate it to get the graph of $y = |x + 3|$.

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct their work.

The student's ALEKS graph:



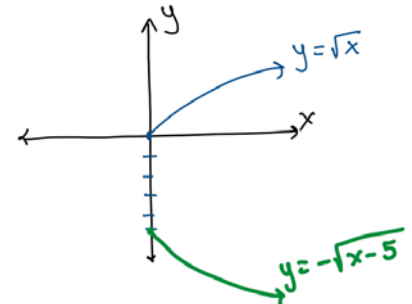
4) The fourth student is working on the following ALEKS problem.

Graph of the function $h(x) = -\sqrt{x - 5}$.

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct their work.

The student wants to enter this graph in ALEKS:



Chapter Learning Objectives

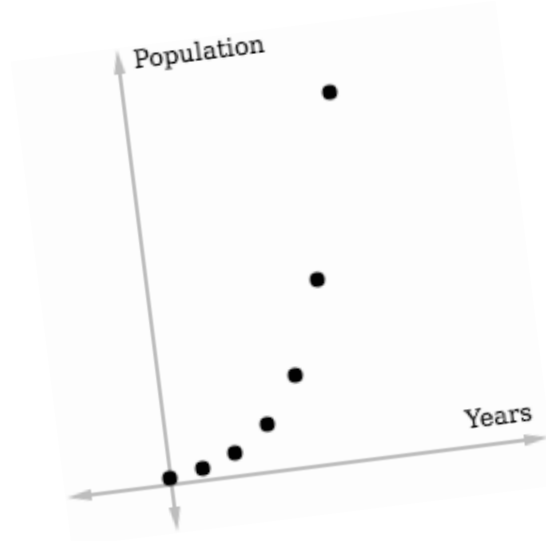
1. Given a real-world situation, decide which type of function should be used to model the data, and justify your choice.
2. Develop the equation of a linear or exponential function that models a given situation.
3. In the context of a given mathematical situation:
 - define input and output variables,
 - interpret solutions, and
 - decide whether a solution is reasonable.
4. From a table of data, write the equation of a linear or exponential function to model the data.
5. Identify and interpret in context the slope and intercepts of a linear model.
6. Interpret inputs and outputs of a model in the context of a given situation.
7. Given the equation of a quadratic model in vertex form, interpret the vertex in context.
8. Given the equation of an exponential model, find and interpret the initial value and growth or decay rate.
9. Determine a reasonable domain and range, in context, of a model.

Chapter 4

How Do We Model Data?

Chapter Overview

Back in Chapter 2, we saw how helpful graphical models can be when analyzing real-world situations. Then, in Chapter 3, our focus was on connections between symbolic and graphical representations of a function. Now, here in Chapter 4, let's put together everything we've discussed so far, to model and analyze real-world situations using symbolic representations of functions.



Chapter 4 Contents

Chapter Learning Objectives	65
Chapter Overview	65
4.1: Warm-Up.....	67
4.1: Selecting a Model Type.....	69
4.2: Algebra Critique – Equations of Linear Functions.....	71
4.3: Writing Equations of Functions to Model Data.....	73
4.4: Interpreting Models in Context.....	79



4.1: Warm-Up

1) Let $g(x) = 3x + 4$.

a) Complete the table of values for $g(x)$:

x	-2	-1	0	1	2
$y = g(x)$					

b) As we increase the input value by 1 unit, how does the output value change?

c) What is the y -intercept? _____

2) Let $h(x) = 4(3)^x$.

a) Complete the table of values for $h(x)$:

x	-2	-1	0	1	2
$y = h(x)$					

b) As we increase the input value by 1 unit, how does the output value change?

c) What is the y -intercept? _____

3) The table of ordered pairs shown at the right defines an exponential function.

x	-1	0	1	2
y	$\frac{1}{8}$	$\frac{1}{2}$	2	8

a) As the input increases by one, how does the output value change?

b) What is the y -intercept? _____

c) Write an exponential function, $y = a(b)^x$, for the table above. _____

- 4) The value, v , in dollars of Sara's smartphone t years after she purchased the phone can be modeled by the function $v(t) = 650 - 150t$.



- a) What is the input variable of this function and what does it represent?
- b) What is the output variable of this function and what does it represent?
- c) Explain using a complete sentence, including units, what the slope represents in context of this situation.
- d) Explain using a complete sentence, including units, what the y -intercept of v represents in the context of this situation.
- e) Explain using a complete sentence, including units, what $v(4)$ represents in the context of this situation.
- f) Determine the coordinates of the t -intercept of $v(t)$.
- g) Interpret your answer to f) in the context of this situation.
- h) What is a reasonable domain for this situation?
- i) What is a reasonable range for this situation?



4.1: Selecting a Model Type

Learning Objectives

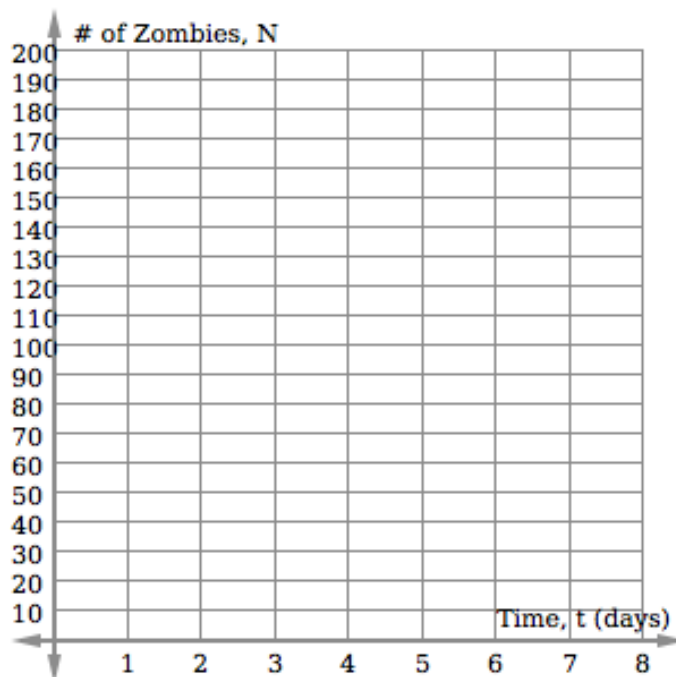
Together with your team:

- Given a “real-world” situation, decide which type of function should be used to model the data, and justify your choice.

- 1) On January 31st, three zombies arrived in Corvallis. On February 1st, they began to bite Corvallis residents and infect the population. Each zombie in the population bites one Corvallis resident, making one new zombie, each day. When they learned of the outbreak on February 2nd, the Center for Disease Control enacted an emergency quarantine measure that prohibited Corvallis residents from leaving the city. The CDC made the following spreadsheet to estimate the zombie population.

Date	Time, t , in days	Number of zombies, N
01-31	0	3
02-01	1	6
02-02	2	12
02-03	3	24
02-04	4	48
02-05	5	96
02-06	6	192

- a) Represent these data graphically, using the axes provided at the right.
- b) What type (family) of function do you think should be used to model the zombie population? Explain.



- c) Could you have determined—without first graphing the data—what type of function should be used to model the zombie population in Corvallis? If so, explain how. If not, explain why not.

- 2) To print shirts with your team logo, a t-shirt printing company charges a one-time set-up fee of \$15.00, and \$8.00 per shirt.

What type of function should be used to model the total charge for printing t-shirts? Explain.



- 3) What is the difference between the two types of functions you selected, the one to model the zombie population and the one to model total charge for printing t-shirts?
- 4) Most days, Katy rides her bike the 1 mile from her home to work. One day, she leaves home riding at a constant pace of 6 mph. After 4 minutes of riding, she arrives at a coffee shop 0.4 mi. from home. She waits 4 minutes for her coffee, then rides quickly to work at 9mph.
- a) Explain *why* neither a linear nor an exponential function can be used to model this situation.
- b) The function that relates Katy's distance, in miles, from home to the elapsed time, in minutes, is an example of a **piecewise-defined** function.

Discuss with your team and write a sentence explaining what you think is meant by the word "piecewise-defined."



4.2: Algebra Critique – Equations of Linear Functions

Learning Objectives

Together with your team:

- Critique the applicability of a mathematical approach or the validity of a mathematical conclusion.
- From the table or graph, or given two points for a linear function, write an equation for the function.

Recall, the slope of a line passing through two points, (x_1, y_1) and (x_2, y_2) is given by the formula,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

A group of Algebra students were working on an ALEKS prep assignment for next week. They are all working on different topics and want to make sure that their answers are valid before entering them into ALEKS. Your task is to decide whether each student's answer is valid or not. **If an answer is invalid, explain why and correct the student's work.**

1) The first student is working on the following ALEKS problem.

● LINES, FUNCTIONS, AND SYSTEMS

Finding slope given two points on the line

Find the slope of the line passing through the points $(-3, -8)$ and $(5, 6)$.

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

The student's work:

$$m = \frac{-3-5}{-8-6} = \frac{-8}{-14} = \frac{4}{7}$$

The student wants to enter in ALEKS:

$$m = \frac{4}{7}$$

2) The second student is working on the following ALEKS problem.

● LINES, FUNCTIONS, AND SYSTEMS

Finding slope given two points on the line

Find the slope of the line passing through the points $(3, 4)$ and $(8, -3)$.

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

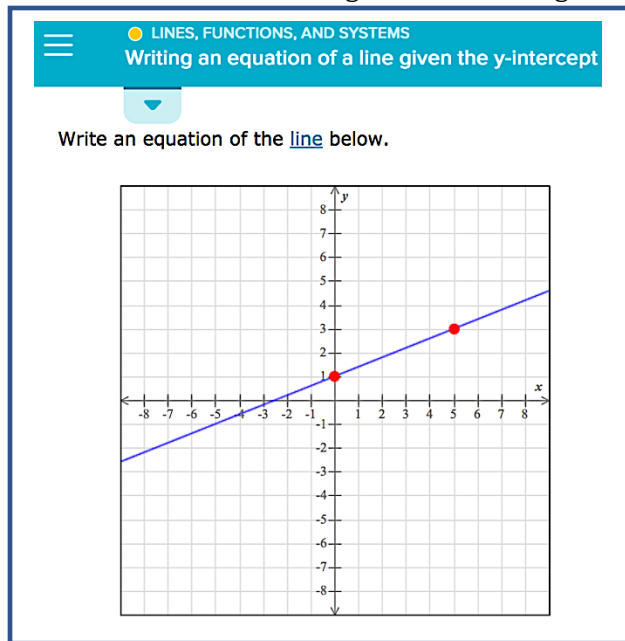
The student's work:

$$m = \frac{4-(-3)}{8-3} = \frac{7}{5}$$

The student wants to enter in ALEKS:

$$m = \frac{7}{5}$$

3) The third student is working on the following ALEKS problem.



The student's work:

$$m = \frac{3 - 1}{5 - 0} = \frac{2}{5}$$

$$y = \frac{2}{5}x + b$$

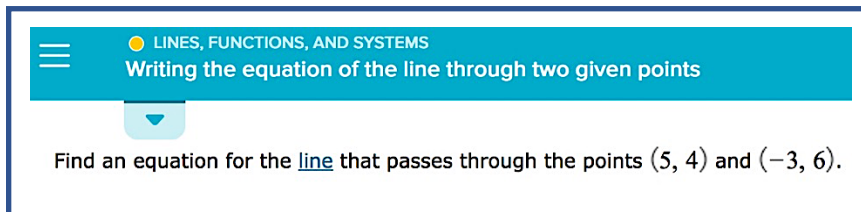
The student wants to enter in ALEKS:

$$y = \frac{2}{5}x + 1$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

4) The fourth student is working on the following ALEKS problem.



The student's work:

$$m = \frac{4 - 6}{5 - (-3)} = \frac{4 - 6}{5 + 3} = \frac{-2}{8} = -\frac{1}{4}$$

$$y = -\frac{1}{4}x + b$$

The student wants to enter in ALEKS:

$$y = -\frac{1}{4}x + 4$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, explain why.



4.3: Writing Equations of Functions to Model Data

Learning Objectives

Together with your team:

- Develop the equation of a linear or exponential function that models a given situation.
- From a table, write a linear or exponential equation to model the data.
- Identify and interpret in context the slope and intercepts of a linear model.
- Interpret inputs and outputs of a model in the context of a given situation (such as stating a reasonable domain and range for the model).

- 1) Consider again the zombie population data from Lesson 4.1.

Date	Time, t , in days	Number of zombies, N
01-31	0	3
02-01	1	6
02-02	2	12
02-03	3	24
02-04	4	48
02-05	5	96
02-06	6	192

- a) What is the input variable and what does it represent?

- b) What is the output variable and what does it represent?

- c) As time increases, how is the zombie population changing?

- d) What is the initial zombie population?

- e) Write the equation of an exponential function, $N(t) = a(b)^t$, that models the number of zombies, N , as a function of time, t , in days.

- f) Use your model and $t = 6$ to verify your model is correct.

- g) According to your model, how many zombies will there be on February 17th?

- h) What is a reasonable domain of the model you wrote in e)? Explain.

Summary: Writing the Equation of an Exponential Function to Model Data Given in a Table

2) Consider again the t-shirt printing scenario from Lesson 4.1. Recall that the company charges a one-time set-up fee of \$15.00, and \$8.00 for each t-shirt printed.

- a) Write the equation of a linear function to model the total cost of printing your team's logo on a t-shirt. (Make sure to first define your input and output variables.)



- b) What is a reasonable domain for the model you wrote in a)? Explain.

- c) According to your model, what is the total cost to print 37 t-shirts?

Summary: Defining a Variable

- 3) Suppose a babysitter charges \$9 per hour to babysit and an additional flat rate of \$6 to cover their transportation costs. She babysits for up to 5 hours at a time. Let's model this situation.

a) What type of function should be used to model this situation? Explain.



b) Define the input; that is, state the variable you will use for your model and what it represents.

c) Define the output; that is, state the variable you will use for your model and what it represents.

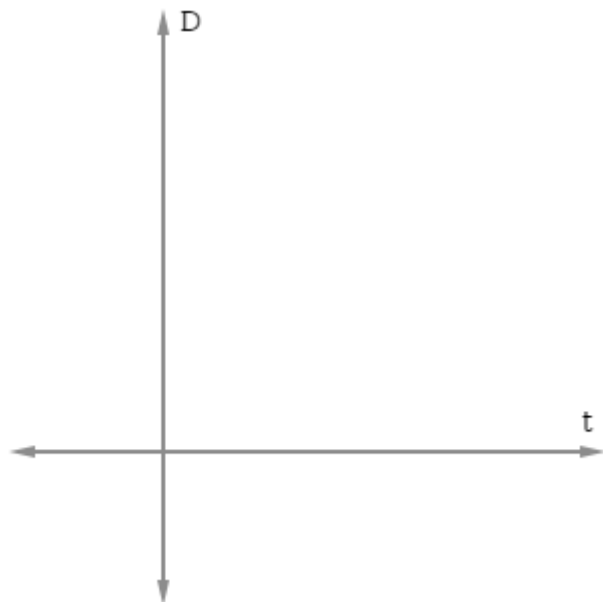
d) Write the equation of a function that models this situation.

- e) What is a reasonable domain for the model you wrote in d)? Explain.
- f) What is a reasonable range for this model? Explain.
- 4) On the first day a new iPhone app was released, it was downloaded by 15 people. The number of downloads tripled every day for the next 9 days. Let's model this situation.
- a) Is this situation best modeled using a linear or exponential function? Justify your answer.
- b) What is the initial number of downloads?
- c) As time increases, how does the number of downloads change? Include units with your answer.
- d) Write an equation to model this situation where the number of downloads, D , is a function of time t .
- e) How many downloads occurred on day 6?

f) Sketch a graph of your model that is good enough to illustrate your answers to b) - e). Be sure to label your axes, indicate the scale on the axes, and label any important points on the graph.

g) What is a reasonable domain of your model, in the context of this situation?

h) What is a reasonable range of your model, in the context of this situation?



5) The cost of tuition at a particular online college varies directly with the number of credits taken.

a) If 11 credits cost \$1375, how much does tuition for 5 credits cost?

b) What type of function should be used to model this situation? Explain.

c) Write an equation of a model relating the number of credits, n and the cost of tuition, C .

d) Interpret each parameter in your equation in the context of this situation. Include units in your answer.

e) How much does it cost to take 16 credits?

- 6) A driver leaves on a trip at 5pm, driving on a long section of open highway at a constant speed. At 5:28pm, he sees a sign that tells him he is 62 miles from his destination. At 5:40pm, his GPS tells him he is 52.4 miles from his destination.
- a) What type of function should be used to model this situation? Explain.
- b) Define the variables you will use when writing a model for this situation. Remember to be specific and include units.
- Inputs: _____ Outputs: _____
- c) Write an equation for a model relating the variables defined in b).
- d) What do you think is a reasonable domain for this model? Explain.
- e) What do you think is a reasonable range for this model? Explain.
- f) According to your model, how far will the driver be from his destination at 5:45pm?
- g) What is the slope of your model from c)? _____ Interpret the slope in the context of the situation. Write your answer as a complete sentence.
- h) What is the y –intercept of the model from c)? _____ Interpret the y –intercept in the context of the situation. Write your answer as a complete sentence.
- i) At what time will the driver reach his destination?



4.4: Interpreting Models in Context

Learning Objectives

Together with your team:

- Interpret in context the coordinates of the y -intercept of a model.
- Identify whether an exponential function is growth or decay.
- Determine the growth or decay rate of an exponential function.
- Determine the coordinates of the y -intercept of any function.

- 1) The value v , in dollars, of a certain car that is t years old can be modeled by the following exponential function.

$$v(t) = 26,000(0.78)^t$$

a) What is the initial value of the car? _____

b) In one sentence, justify your answer to a).

- c) Continue the pattern to complete the following table of values for $v(t)$. You do not need to calculate the output each time.

Input	Output Calculation
$t = 0$	$v(0) = 26,000(0.78)^0$
$t = 1$	$v(1) = 26,000(0.78)^1$
$t = 2$	$v(2) = 26,000(0.78)^2 = 26,000(0.78)^1 \cdot (0.78)$
$t = 3$	$v(3) = 26,000(0.78)^3 = 26,000(0.78)^2 \cdot (0.78)$
$t = 4$	
$t = 5$	
$t = 6$	

- d) Is the value of the car increasing or decreasing over time? INCREASING DECREASING

e) Use the table values to determine by what *percentage* the value of the car changes each year.

Summary: Interpreting Exponential Models of the Form, $y = a(b)^x$, with $b > 0$, $b \neq 1$ and $a > 0$		
	Exponential Growth	Exponential Decay
Definition		
Percent Change		

- 2) A child's grandmother set up a savings account for her when she was born, but has not put any money in the account since then. The amount of money A , in dollars, in the account after t years can be modeled by the function:

$$A(t) = 200(1.02)^t$$

- How much money did the child's grandmother put in the account when she was born?
- Is the amount of money in the account over time modeled by exponential growth or decay? Explain.
- By what percentage does the amount of money in the account change each year?
- How much money will be in the account when the child turns 18?

- 3) In an engineering competition, student teams design a device that can launch a water balloon across a field. The team whose balloon attains the maximum height wins the contest.

Using a computer program, Elizabeth's team has determined a model for the *path* of a water balloon in their launcher:

$$h(t) = -0.5(t - 8.5)^2 + 45.125,$$

where h is the height, in feet, of the water balloon above the ground at time t , in seconds.

- a) What is the maximum height of the water balloon (include units in your answer)?

- b) After how many seconds will their water balloon reach this maximum height?

- 4) At the beginning of 2016, an investor purchased stock in a small company. The value of the stock purchase can be modeled by the function $v(t) = (t - 5)^2 + 725$, where v is the value of the stock purchase, in dollars, after t months for $0 \leq t \leq 12$.

- a) Will the value of the stock purchase have a minimum/maximum value? Explain your reasoning.

- b) When will the value of the stock have a minimum/maximum value (include units)?

- c) What is the minimum or maximum value for the stock purchase (include units)?

- d) Summarize your answers from a) – c) in a complete sentence.

Chapter Learning Objectives

1. Use symbolic methods to solve linear equations and inequalities, and systems of linear equations, and to check solutions.
2. Relate the solution(s) of an equation, inequality, or system of equation(s), to points on the graph of the corresponding function(s), and interpret in context.
3. Given the equation of a function, determine the coordinates of its intercept(s), if any.
4. Convert the equation of a quadratic function from:
 - vertex to standard form;
 - factored form to standard form and, if possible, vice versa; and
 - factored form to vertex form.
5. Use the discriminant to determine the number of solutions to a quadratic equation.
6. Solve quadratic equations using factoring, the quadratic formula, and the square root property.
7. Solve absolute value equations by applying the definition of absolute value as the distance between a number and zero.
8. Develop and interpret models to solve problems presented in the context of real-world situations.

Chapter 5

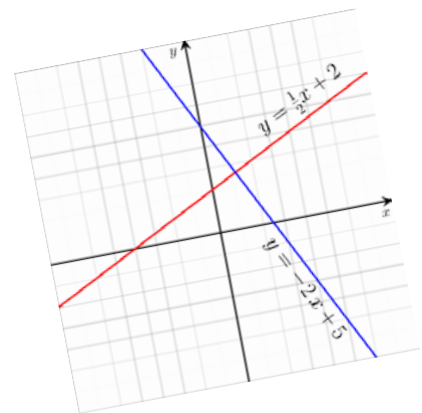
What Can We Learn from Equations?

Chapter Overview

Equations can help us answer many important questions about the functions they represent, such as:

- What is the value of the function for a certain input?
- For which inputs does the function have an output equal to/greater than/less than a particular value?
- When will the function reach 0, if ever?
- How many times do two functions take on the same value, and for which inputs does this occur?

In this final chapter, although our focus will be on symbolic methods for solving equations, inequalities, and systems of equations, we will continue to look for connections between symbolic and graphical function representations. By making these connections explicit, we strive to deepen conceptual understanding of functions, as well as help to strengthen algebraic reasoning abilities.



Chapter 5 Contents

Chapter Learning Objectives	83
Chapter Overview	83
5.1: Warm-Up.....	85
5.1: Solving Linear Equations and Inequalities	87
5.2: Warm-Up.....	95
5.2: Solving Systems of Linear Equations	96
5.3: Warm-Up.....	103
5.3: Solving Absolute Value Equations	104
5.4: Algebra Critique – Factoring.....	107
5.5: Different Symbolic Forms of Quadratic Functions	109
5.6: Warm-Up.....	117
5.6: Solving Quadratic Equations & Modeling.....	119
5.7: Algebra Critique – Simplifying Rational Expressions.....	125



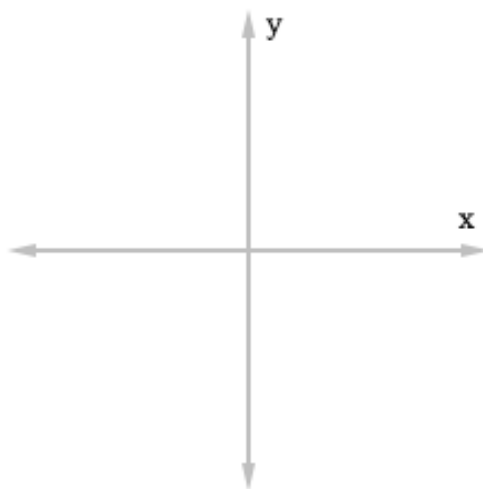
5.1: Warm-Up

1) Let $f(x) = 2x - 5$.

a) Evaluate: $f(0)$

b) Solve the equation for x : $f(x) = 0$.

c) Sketch a graph of $f(x) = 2x - 5$ that's good enough to illustrate your answers for a) and b).



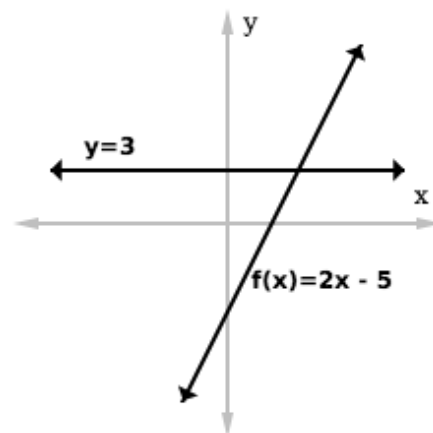
Summary: Finding intercepts of a function, given its equation

y-intercept	x-intercept(s)

2) Again, consider the function $f(x) = 2x - 5$.

a) Solve the equation for x : $f(x) = 3$.

b) Explain how the graph is related to the equation you solved in a).



3) A student was asked to solve the following equation for x .

$$19x - 1 + 5x = 2(7x - 3) + 11$$

The student found the solution is $x = 3$.

Is the student correct? Explain.



5.1: Solving Linear Equations and Inequalities

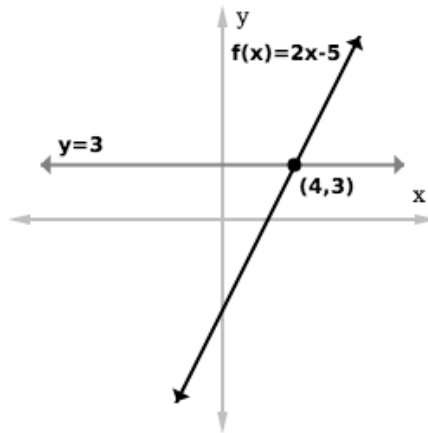
Learning Objectives

Together with your team:

- Develop and interpret models to solve problems presented in the context of real-world situations
- Use symbolic methods to solve linear equations and inequalities.
- Check solutions to equations and inequalities.
- Relate the solution(s) of an equation or inequality to point(s) on the graph of the corresponding function(s), and interpret in context.

- 1) A team of teachers won a grant of \$4000 to improve their math classroom. They plan to purchase a set of tablets for students to use during class. After some internet research, they found the best price on tablets is \$254.12 each. The total delivery charge will be \$98.00. How many tablets could the teachers buy using their grant funds?
 - a) Define the input and output variables you will use for your model and what each variable represents.
 - b) Write the equation of a function that models this situation using the variables you defined in a).
 - c) Write an *inequality* you could use to solve for the possible number of tablets the teachers could buy using their grant funds.
 - d) Solve the inequality you wrote in c). (Hint: think about the domain of the function when writing your solution.)
 - e) Interpret your solution from d) in the context of this situation. Write a complete sentence.

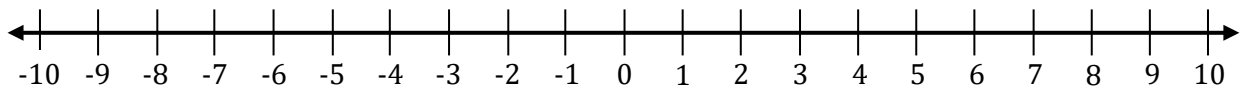
- 2) In the 5.1 Warm Up, when solving the equation $2x - 5 = 3$, we found the solution $x = 4$. One way to represent this solution graphically is as follows:



- a) Explain how the graph above is related to the solution of the *inequality* $2x - 5 > 3$.

- b) Another way to graphically represent the solution set of an inequality is with a number line.

Using the number line below, sketch the solution set of the inequality $2x - 5 > 3$.



- 3) Consider the linear function: $g(x) = -4x + 13$.

- a) Solve the inequality: $-4x + 13 \geq 15$
- b) Suppose a couple of your teammates are not convinced your answer to a) is correct. Below include graphs, equations, and/or number lines you think would help them understand why you are right.

Summary: Solving Linear Inequalities (Test-point Method)

1. Solve the corresponding *equation* first (with an = sign.)
2. Solve the *inequality*, by testing a value on one side of (greater or less than) the solution value found in Step 1.

If the inequality is *true* at the test value, then _____

If the inequality *false* at the test value, then _____

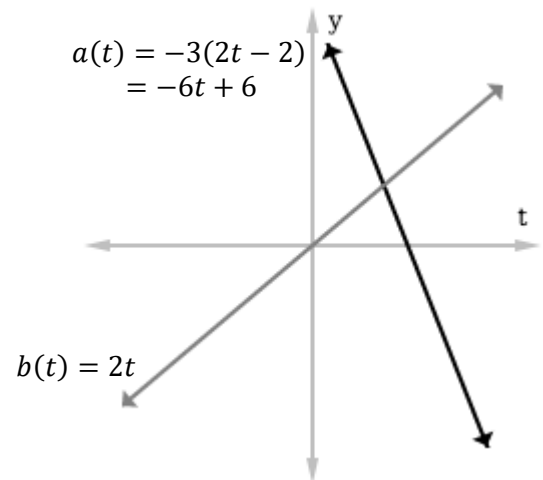
3. Check your solution; for instance, another representation may be helpful in deciding whether your answer makes sense.

In **1) – 3)**, we compared a linear function to a numerical value, $y = 3$, (a constant function). Next, let's compare two *non-constant* linear functions.

- 4)** Consider the linear functions graphed at the right:

$$a(t) = -3(2t - 2) \text{ and } b(t) = 2t$$

- a) Solve the equation for t : $a(t) = b(t)$.



- b) Where is the solution you found in a) represented in the graph?

- c) Solve the inequality for t :

$$-3(2t - 2) \geq 2t$$

- d) Check your solutions to a) and c).

5) Use the given inequality to answer each part below.

$$-4u - 25 \leq 2u + 17$$

a) Solve for u .

b) Check your solution.

c) Represent your solution from a) graphically (on a number line or graph of the two linear functions).

d) Write your solution in inequality notation: _____ and in interval notation: _____

- 6) A classmate was working on solving linear equations in ALEKS and brought their notebook work to show you. They had tried solving three very similar equations, but came up with different types of answers.

Consider the student's work below on the three questions.

Q1.

$$\begin{aligned}
 3(x-2) - 2x &= x-6 \\
 3x-6-2x &= x-6 \\
 x-6 &= x-6 \\
 -x &\quad -x \\
 -6 &= -6 \\
 +6 &\quad +6 \\
 0 &= 0 \\
 \text{zero does equal zero...}
 \end{aligned}$$

Q2.

$$\begin{aligned}
 3(x-1) - 2x &= x-6 \\
 3x-3-2x &= x-6 \\
 x-3 &= x-6 \\
 -x &\quad -x \\
 -3 &= -6 \leftarrow \text{I don't know what this means?}
 \end{aligned}$$

Q3.

$$\begin{aligned}
 3(x-1) - x &= x-6 \\
 3x-3-x &= x-6 \\
 2x-3 &= x-6 \\
 -x &\quad -x \\
 x-3 &= -6 \\
 +3 &\quad +3 \\
 \boxed{x = -3}
 \end{aligned}$$

- a) Help your classmate make sense of the three solutions above. How could you use a graph?

Q1.

Q2.

Q3.

- b) What should your classmate enter for each answer?

Q1.

Q2.

Q3.

Summary: Solutions to Linear Equations	
Conclusion	This means...
Contradiction (such as: _____)	The equation has _____ solutions.
A linear equation (such as: _____)	The equation has _____ solution.
Identity (such as: _____)	_____ are solutions.

7) Consider the linear equation given below:

$$\frac{3x}{2} + 7 = \frac{x}{9} - 11$$

a) What would be your first step in solving for x ? Why?

b) Solve the equation for x . Simplify your answer as much as possible.

$$\frac{3x}{2} + 7 = \frac{x}{9} - 11$$

c) Check your solution.

Summary: Solving Equations Involving Fractions



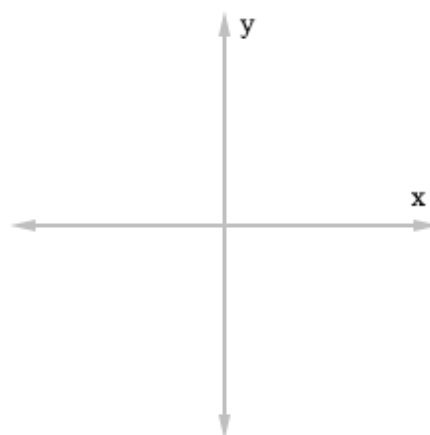
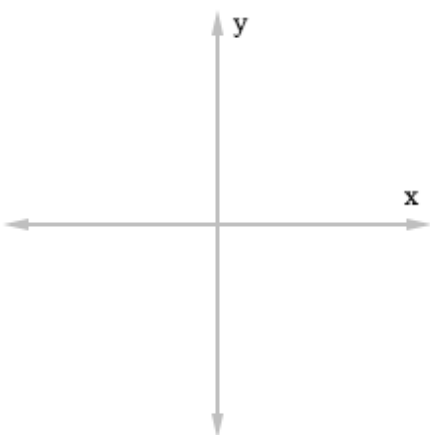
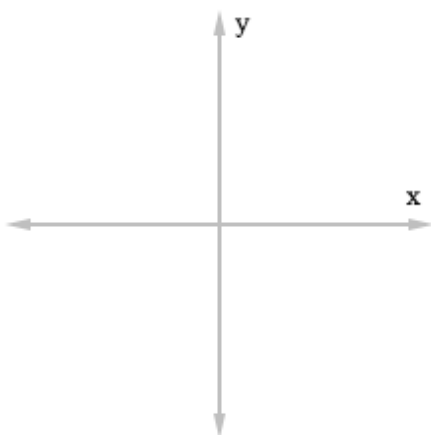
5.2: Warm-Up

1) On each set of axes below, sketch a graph of two lines with the given property.

a) Two lines that do not intersect.

b) Two lines that intersect in a single point.

c) Two lines that intersect in infinitely many points.



2) A student was asked to solve the following system of equations.

$$2x - 3y = -1$$

$$8x - 6y = -4$$

The student found the solution $(x, y) = (1, 1)$.

Is the student correct? Explain.



5.2: Solving Systems of Linear Equations

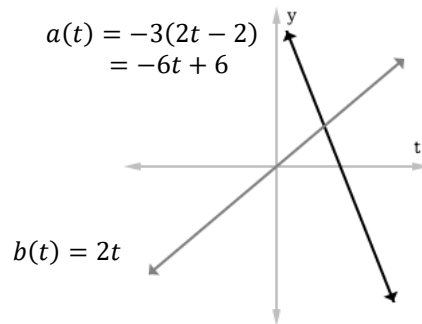
Learning Objectives

Together with your team:

- Develop and interpret models to solve problems presented in the context of real-world situations
- Use symbolic methods to solve systems of linear equations.
- Relate the solution(s) of a system of equations to point(s) on the graph of the corresponding functions, and interpret in context.

Next, we turn our attention to solving **systems of linear equations**.

- 1) Recall, in Lesson 5.1, we solved the linear equation, $-3(2t - 2) = 2t$, and related the solution to the graph of the corresponding linear functions.



Considered together, these two functions are called a “system.”

To **solve a system of linear equations** means, to find the (input, output) coordinate pairs for the point(s) where the lines intersect, if any.

- a) Solve the system of linear equations:

$$y = -3(2t - 2)$$

$$y = 2t$$

- b) Where is the solution you found in a) represented in the graph?

- c) Check your solution to a).

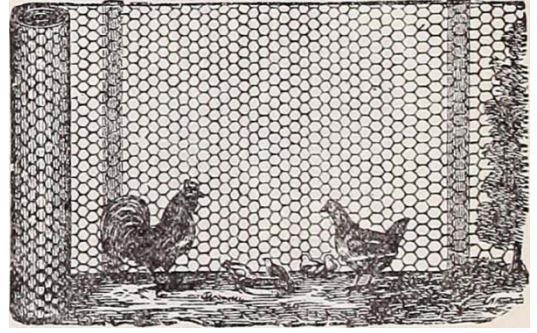
- 2) A shopper is comparing two cell phone data plans.
- AT&T has a plan that charges \$50 per GB of data each month.
 - Verizon has a plan that charges \$90 per month for unlimited data.

The shopper wants to know which plan is a better deal, but realizes it depends on how many GB are used each month.

- a) Define two variables in this situation. State what the variables represent and include units.
- b) For each data plan, write the equation of a linear function to model the cost of data for that plan; in other words, use the variables you defined in a) to write a system of linear equations to represent this situation.
- c) How many GB of data will result in the two plans costing the same?
- d) Your data-conscious friend uses approximately 0.9 GB of data per month. Which plan would be more economical for them? Explain.
- e) However, you use approximately 3.5 GB of data per month. Which plan would be more economical for you? Explain.

- 3) A farmer has a 330-foot roll of fencing wire to use for making a rectangular chicken pen. If the farmer wants to make the length of the rectangle 1.5 times longer than the width, what should be the length of each side?

a) Draw a picture of the chicken pen and label it.



b) Define two variables to represent the unknown quantities in this situation. Make sure to include units.

c) Using the variables you defined above, write a system of linear equations representing this situation.

d) Solve your system of equations.

e) Write a complete sentence interpreting your solution to d) in the context of this situation.

- 4) Two groups of students take a break from studying. Someone from each group volunteers to go get drinks for everyone, as long as each person pays them back for their drink.

- Group A orders four bottles of water and two mochas.
- Group B orders two bottles of water and three mochas.



When they get back with the drinks, they realize the receipt does not show the price of individual drinks, only the total. Group A's order came to \$12.30 and Group B's was \$13.33.

- Define two variables to represent the unknown quantities in this situation. State what the variables represent and include units.
- Write a system of equations representing this situation.
- Solve your system of equations.
- Write a complete sentence interpreting your answer to c) in the context of this situation.

- 5) A classmate was working on solving systems of linear equations in ALEKS and brought their notebook work to show you. They had tried solving two very similar systems of equations, but came up with different types of answers.

Consider the student's work below.

Q1.

$$\begin{aligned}
 4x - 10 &= 2y \\
 2x - y &= 5 \rightarrow 2x - 5 = y \\
 4x - 10 &= 2(2x - 5) \\
 4x - 10 &= 4x - 10 \\
 -4x &\quad -4x \\
 -10 &= -10 \leftarrow \text{Does this mean} \\
 &\quad \text{the solution is } -10?
 \end{aligned}$$

Q2.

$$\begin{aligned}
 4x + 8 &= 2y \\
 -2x + y &= -16 \rightarrow y = 2x - 16 \\
 4x + 8 &= 2(2x - 16) \\
 4x + 8 &= 4x - 32 \\
 -4x &\quad -4x \\
 8 &= -32 \leftarrow \text{This seems} \\
 &\quad \text{wrong}
 \end{aligned}$$

- a) Help your classmate make sense of the two solutions above. How could you use a graph?

Q1.

Q2.

- b) What should your classmate enter for each ALEKS answer?

Q1.

Q2.

Summary: Solutions to Systems of Linear Equations

Type of system	This means...	The graphs of the lines are...
Inconsistent system	The system has _____ solutions.	
Consistent independent system	The system has _____ solution.	
Consistent dependent system	The system has _____ solutions.	

For each of the remaining systems of linear equations in this lesson, solve the system, then check your solution.

6)
$$\begin{aligned} 4x + 3y &= 4 \\ 4x + 6y &= 16 \end{aligned}$$

7)
$$\begin{aligned} -2x + y &= -9 \\ 5x - 4y &= 24 \end{aligned}$$

8)
$$\begin{aligned} 4x + 8 &= 2y \\ -2x + y &= -16 \end{aligned}$$

9)
$$\begin{aligned} 4x - 10 &= 2y \\ 2x - y &= 5 \end{aligned}$$

10)
$$\begin{aligned} 5x + 6y &= -10 \\ 5x + 3y &= 20 \end{aligned}$$

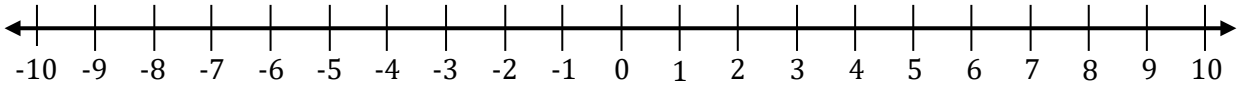
11)
$$\begin{aligned} y &= 2x + 1 \\ 5x - 4y &= -7 \end{aligned}$$



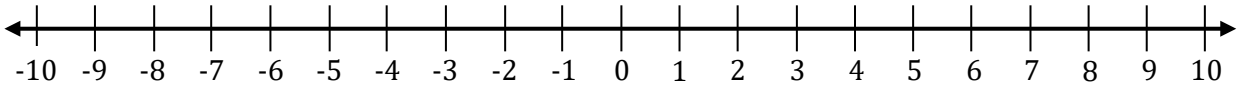
5.3: Warm-Up

- 1) For each distance statement given below, fill in the blank(s), then represent the statement on the number line provided.

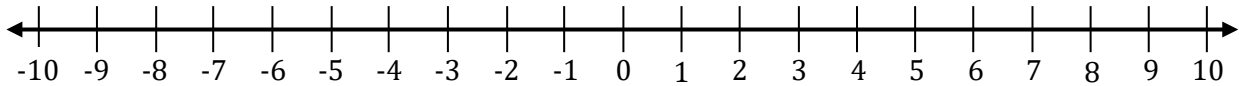
- a) The distance between 2 and 0 is _____.



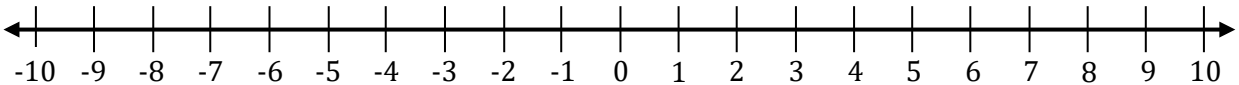
- b) The distance between -2 and 0 is _____.



- c) $|-7|$ represents the distance between _____ and _____.



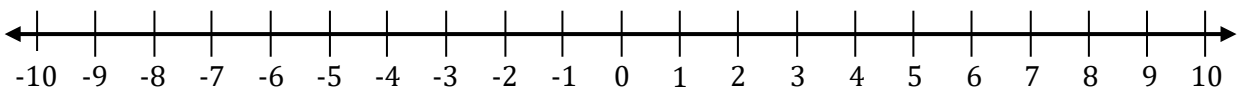
- d) $|x|$ represents the distance between _____ and _____.



- e) $|x| = 5$ means the distance between _____ and _____ is _____.



- f) If the distance between x and 0 is 9, then x could be _____ or _____.





5.3: Solving Absolute Value Equations

Learning Objectives

Together with your team:

- Solve absolute value equations by applying the definition of absolute value as the distance between a number and zero.

1) Solve each of the following equations for the specified variable. Then check your solution(s).

Hint: it may be helpful to translate absolute value statements into words, as in the 5.3 Warm-up.

<p>a) If $x - 18 = -10$, then $x =$ _____</p> <p>Check:</p>	<p>b) If $-2x = 20$, then $x =$ _____</p> <p>Check:</p>
<p>c) If $t + 11 = 11$, then $t =$ _____</p> <p>Check:</p>	<p>d) If $u + 4 = 3$, then $u =$ _____</p> <p>Check:</p>
<p>e) If $4 w = 6$, then $w =$ _____</p> <p>Check:</p>	<p>f) If $x + 1 = 8$, then x _____</p> <p>Check:</p>

2) When solving the absolute value equation below, what must be your first step?

$$-5|x + 1| = -20$$

- A. Set $x + 1$ equal to 20 or -20 .
- B. Distribute the -5 inside the absolute value sign.
- C. Isolate the absolute value by dividing both sides by -5 .
- D. There's no need to do a first step because an absolute value can't be equal to a negative number, so the equation has no solution.

Summary: Solving Absolute Value Equations

No Solution	One Solution	Two Solutions

Solve the following absolute value equations. Be sure to check your solutions!

3) $-2|z - 17| + 10 = 3$

4) $|4u - 40| + 20 = 13$

5) $5|u + 6| - 57 = -7$

6) $-3|4 - t| + 5 = 5$



5.4: Algebra Critique – Factoring

Learning Objectives

Together with your team:

- Critique the applicability of a mathematical approach or the validity of a mathematical conclusion.
- Factor quadratic functions using various techniques.

A group of Algebra students were working on an ALEKS prep assignment for next week. They are all working on different topics and want to make sure that their answers are valid before entering them into ALEKS. Your task is to decide whether each student's answer is valid or not. **If an answer is invalid, explain why and correct the student's work.**

1) The first student is working on the following ALEKS problem.

☰
● EXPONENTS, POLYNOMIALS, AND FACTORING

Factoring out a monomial from a polynomial: Univariate

▼

Factor $15a - 6a^2$.

The student's work:

$$15a - 6a^2$$

$$a(15 - 6a)$$

The student wants to enter into ALEKS:

$$a(15 - 6a)$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

2) The second student is working on the following ALEKS problem.

☰
● EXPONENTS, POLYNOMIALS, AND FACTORING

Factoring a quadratic with leading coefficient 1

▼

Factor.

$$x^2 + 10x - 24$$

The student's work:

$$x^2 + 10x - 24$$

$$(x + 6)(x + 4)$$

The student wants to enter into ALEKS:

$$(x + 6)(x + 4)$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

3) The third student is working on the following ALEKS problem.

● EXPONENTS, POLYNOMIALS, AND FACTORING

Factoring a quadratic with leading coefficient greater than 1:

Factor.

$2z^2 - 9z + 7$

The student's work:

$$2z^2 - 9z + 7$$

$$(2z - 7)(z - 1)$$

The student wants to enter in ALEKS:

$$(2z - 7)(z - 1)$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

4) The fourth student is working on the following ALEKS problem.

● EXPONENTS, POLYNOMIALS, AND FACTORING

Factoring a difference of squares in one variable: Basic

Factor.

$w^2 - 25$

The student's work:

$$w^2 - 25$$

$$(w - 5)^2$$

The student wants to enter in ALEKS:

$$(w - 5)^2$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, explain why.



5.5: Different Symbolic Forms of Quadratic Functions

Learning Objectives

Together with your team, given the equation of a quadratic function:

- Convert between different forms of a quadratic function (vertex form, standard form, and factored form).

- 1) Recall, in Chapter 3, we explored the influence of the parameters a , h , and k on the graph of a quadratic function given in **vertex form**, $y = a(x - h)^2 + k$.

- a) The parameter a is called the _____ of the quadratic.

Recall, if $a > 0$, then the graph of the quadratic will _____.

And if $a < 0$, then the graph of the will _____.

- b) The coordinates of the vertex of the parabola are: _____

- 2) In addition to the vertex form, there are two other common forms for representing a quadratic function symbolically: **standard form** and **factored form**.

- Standard form: $y = ax^2 + bx + c$
- Factored form: $y = a(x - r_1)(x - r_2)$

- a) For each quadratic function given in the table below, decide whether it is written in vertex, standard, or factored form. Then write the values of the parameters under the correct form.

	Vertex form $y = a(x - h)^2 + k$			Standard form $y = ax^2 + bx + c$			Factored form $y = a(x - r_1)(x - r_2)$		
Quadratic function	a	h	k	a	b	c	a	r_1	r_2
$y = -2x^2 + 9x - 5$									
$y = -3(x + 5)^2 - 6$									
$y = \frac{1}{2}(x - 5)(x + 2)$									

- b) When a quadratic function is written in vertex form, it is easy to identify the coordinates of its vertex, and whether the parabola opens upward or downward.

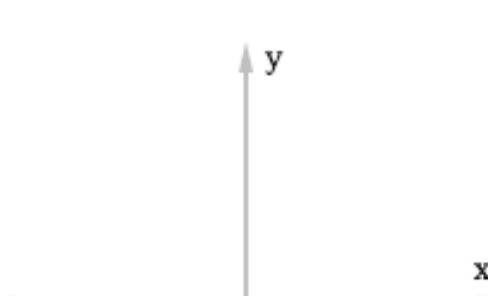
For each of the other two forms, what features of the graph can you determine—at a glance—about the graph of the parabola? Hint: if you are not sure, try exploring in Desmos, using the functions in a).

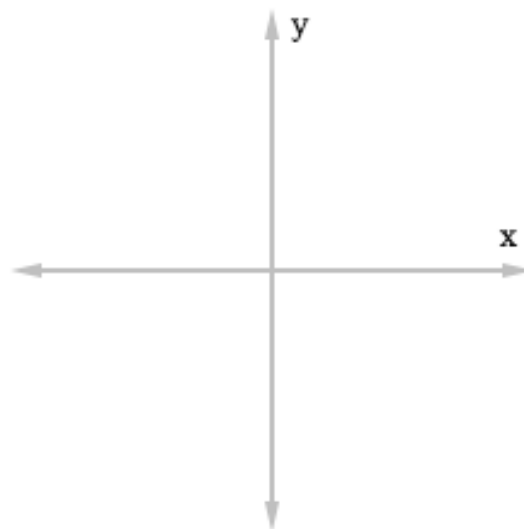
	Features of the graph from the equation
Standard form: $y = ax^2 + bx + c$	
Factored form: $y = a(x - r_1)(x - r_2)$	

Because different symbolic forms give us different information about the graph of a quadratic function, we may wish to convert one form to another.

3) Let g be the quadratic function given below.

$$g(x) = -2(x + 3)^2 - 7$$

- a) Rewrite $g(x)$ in standard form.
- b) Write a sentence describing how to convert a quadratic function from **vertex form to standard form**.
- c) What is the vertex of the graph of $y = g(x)$?
- d) What is the y -intercept of the graph of $y = g(x)$?
- e) Does the parabola open upward or downward?
- f) Sketch a graph of g that's good enough to illustrate your answers to c) – e).
- 
- A blank Cartesian coordinate system is shown in the bottom right corner of the page. It consists of a horizontal x-axis and a vertical y-axis, both represented by gray lines with arrows at their ends. The x-axis is labeled with a bold 'x' at its right end, and the y-axis is labeled with a bold 'y' at its top end. The origin is the point where the two axes intersect.



4) Let w be the quadratic function given below.

$$w(x) = 4x^2 - 12x - 40$$

- a) Rewrite $w(x)$ in factored form.

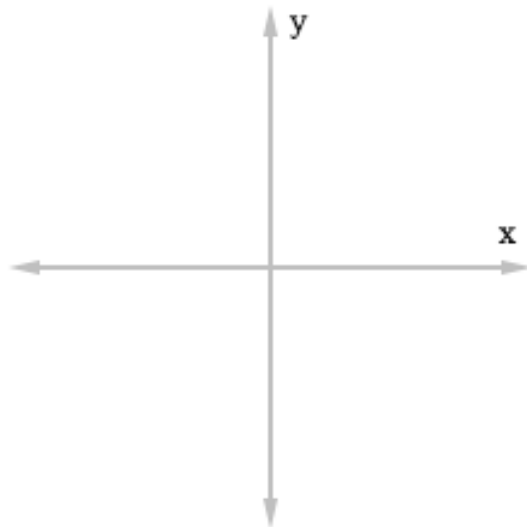
- b) Write a sentence describing how to convert a quadratic function from **standard form to factored form**.

- c) What is the y -intercept of the graph of $y = w(x)$?

- d) What are x -intercept(s) of the graph of $y = w(x)$?

- e) Does the parabola open upward or downward?

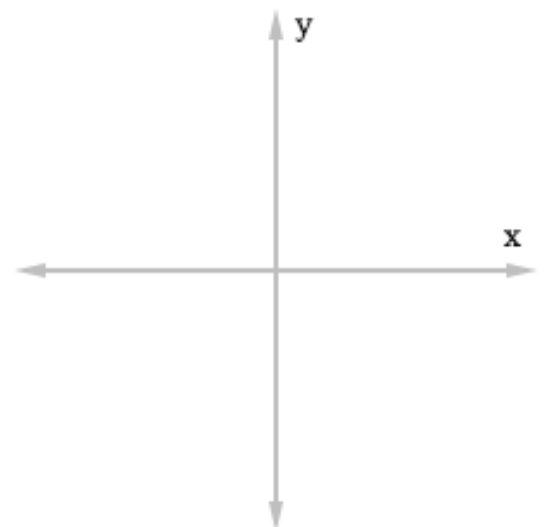
- f) Sketch a graph of g that's good enough to illustrate your answers to c) – e).



5) Let h be the quadratic function given below.

$$h(x) = 4(2x + 1)(x - 3)$$

- a) Rewrite $h(x)$ in standard form.
- b) Write a sentence describing how to convert a quadratic function from **factored form to standard form**.
- c) What are the x -intercepts of the graph of $y = h(x)$?
- d) What is y -intercept of the graph of $y = h(x)$?
- e) Does the parabola open upward or downward?
- f) Sketch a graph of g that's good enough to illustrate your answers to c) – e).



6) Let $y = p(x)$ be a quadratic function with x -intercepts/zeros at $x = -8$ and 3 .

a) What are the factors of the function? Explain.

b) Where is the axis of symmetry of the graph of $y = p(x)$? Hint: sketch a graph.

7) Let m be the quadratic function given in factored form below.

$$m(x) = -3(x - 4)(x + 6)$$

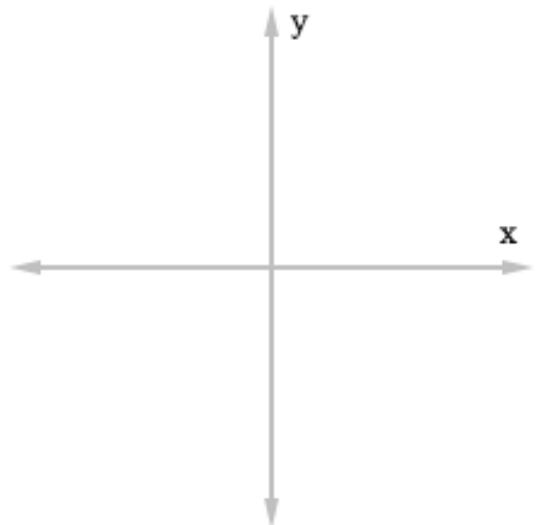
a) What are the x -intercepts of the graph of $y = m(x)$?

b) What is the equation of the axis of symmetry of the graph of $y = m(x)$?

c) What is the x -coordinate of the vertex of the graph of the parabola? Explain.

d) Does the parabola open upward or downward? Explain.

e) Sketch a graph of m that's good enough to illustrate your answers to a) – d). *Draw the axis of symmetry on your graph as a dotted line.*



- f) In order to write $m(x) = -3(x - 4)(x + 6)$ in vertex form, $m(x) = a(x - h)^2 - k$, we need the values of three parameters. What are these three parameters?
- g) How can you determine the y -coordinate of the vertex of the parabola you graphed in e)?
- h) Determine the values of the three parameters you listed in f) and write the equation of $y = m(x)$ in vertex form.
- i) List the steps describing how to convert a quadratic function from **factored form to vertex form**.

- 8) A water balloon is launched across a field. The path of the water balloon can be modeled by a quadratic function, height = $h(t)$, where t is the elapsed time in seconds, and $h(t)$ is the height of the balloon above the ground, in feet.

$$h(t) = -0.5t^2 + 8.5t + 9 \quad \text{Standard form}$$

$$h(t) = -0.5(t + 1)(t - 18) \quad \text{Factored form}$$

$$h(t) = -0.5(t - 8.5)^2 + 45.125 \quad \text{Vertex form}$$

Use the equations above to answer the following questions.

- a) Which form of the quadratic function gives you the *initial height* of the balloon?

Standard

Factored

Vertex

- b) Which form of the quadratic function gives you the *maximum height* of the balloon?

Standard

Factored

Vertex

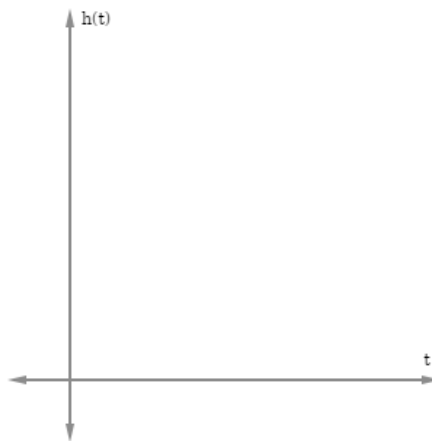
- c) Which form of the quadratic function tells you when the balloon *hit the ground*?

Standard

Factored

Vertex

- d) Sketch a graph of the situation that's good enough to illustrate the initial height and maximum height of the balloon and shows when the balloon hit the ground. Label the important points!



- e) Write a sentence summarizing what your graph shows. Make sure to include units in your answer.



5.6: Warm-Up

For each quadratic equation given below, decide whether is possible to solve the equation using the Quadratic Formula, Square Root Property, and/or Factoring with the Zero Product Property. For each method, place a check mark under the best description.

- ✓ A fine method (or the only method) for solving the equation.
- ✓ Maybe you could use the method, but there is another method that's better.
- ✓ It is not possible to use the method to solve the equation.

Equation to solve:	Quadratic Formula: If $0 = ax^2 + bx + c$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$			Square Root Property: If $x^2 = a$, then $x = \pm\sqrt{a}$			Factor/ Zero Product Property: If $a \cdot b = 0$, then $a = 0$ or $b = 0$		
	A fine method	Maybe, but there's a better method	Not possible to use this method	A fine method	Maybe, but there's a better method	Not possible to use this method	A fine method	Maybe, but there's a better method	Not possible to use this method
$0 = x^2 - 7x + 10$									
$(x + 3)^2 + 2 = 83$									
$-(x - 4)^2 - 3 = 6$									
$(x + 4)(3x - 2) = 0$									
$4x^2 = 9$									
$2x^2 - 6x = 3$									
$3x^2 - 15x = 0$									
$10x^2 + 13x - 3 = 0$									
$5(x - 3)(2x - 1) = 0$									
$-5x^2 = 10x$									

Summary: Methods for Solving Quadratic Equations



5.6: Solving Quadratic Equations & Modeling

Learning Objectives

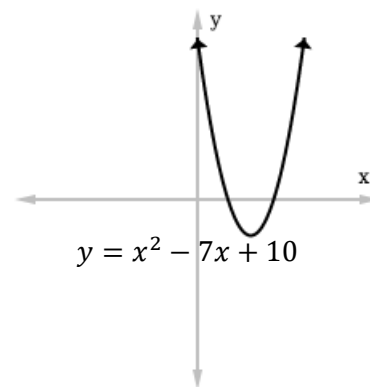
Together with your team:

- Solve quadratic equations by factoring, using the square root property, and applying the quadratic formula, and check solutions to equations numerically.
- Relate the solution(s) of an equation to point(s) on the graph of the corresponding function(s), and interpret in context.
- Utilize the discriminant to determine the number of solutions to a quadratic equation.

1) Next, using your preferred method from the 5.3 Warm-up, solve each of the following quadratic equations for x . Then, check your solutions and indicate in the given graph where the solutions are represented.

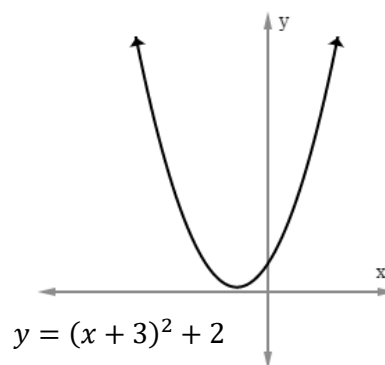
a) $0 = x^2 - 7x + 10$

Check:



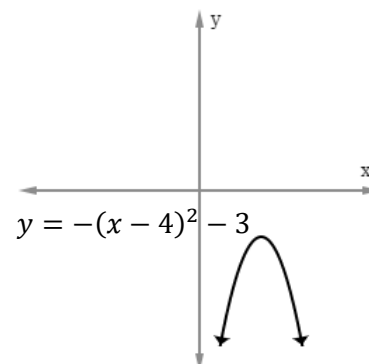
b) $(x + 3)^2 + 2 = 83$

Check:



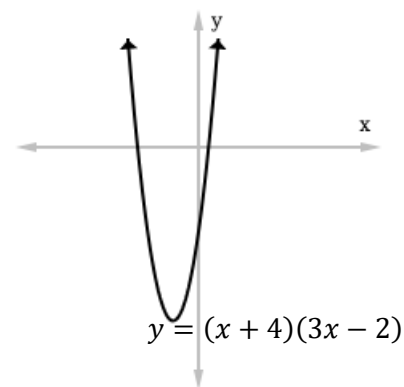
c) $-(x - 4)^2 - 3 = 6$

Check:



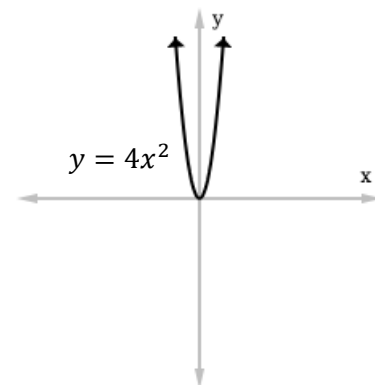
d) $(x + 4)(3x - 2) = 0$

Check:



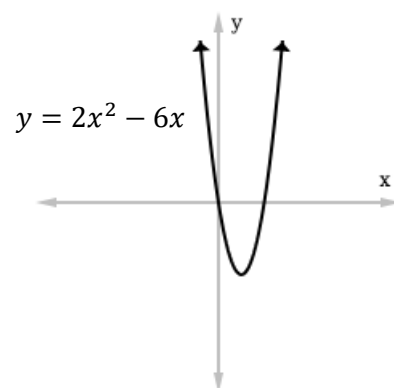
e) $4x^2 = 9$

Check:



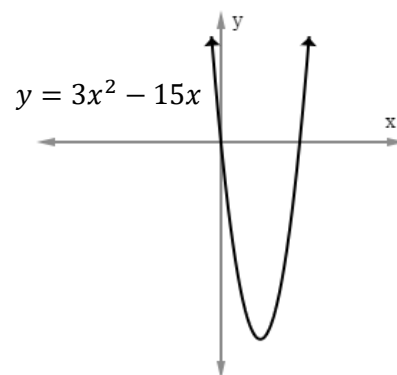
f) $2x^2 - 6x = 3$

Check:

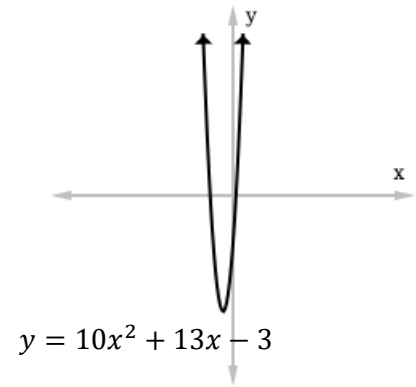


g) $3x^2 - 15x = 0$

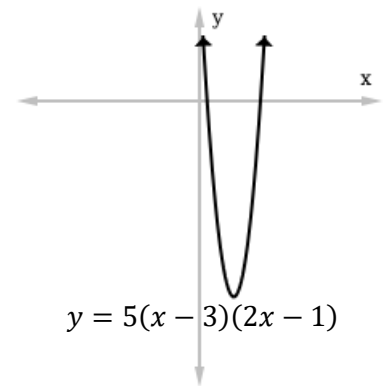
Check:



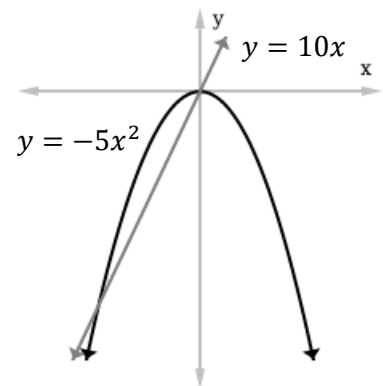
h) $10x^2 + 13x - 3 = 0$ Check:



i) $5(x - 3)(2x - 1) = 0$ Check:



j) $-5x^2 = 10x$ Check:



Summary: Solving Quadratic Equations			
	No Solution	One Solution	Two Solutions
Quadratic Formula: If $0 = ax^2 + bx + c$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$			
Square root property: If $x^2 = a$, then $x = \pm\sqrt{a}$			
Factor/Zero Product Property: If $a \cdot b = 0$, then $a = 0$ or $b = 0$			

- 2) Notice the “No Solution” box is greyed out for Factoring/Zero Product Property. When a quadratic does not factor, why can we NOT conclude the quadratic has “no solution”?



- 3) A small local movie theater can model their daily revenue, R , as a function of ticket price, p , with the following quadratic function, $y = R(p)$.

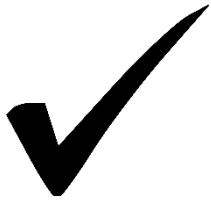
a) All three forms of the function are given. Label each form standard, factored or vertex.

$$R(p) = -80p^2 + 1400p \quad \underline{\hspace{10em}}$$

$$R(p) = -80p(p - 17.5) \quad \underline{\hspace{10em}}$$

$$R(p) = -80(p - 8.75)^2 + 6125 \quad \underline{\hspace{10em}}$$

- b) What is the input variable? What does it represent in this context?
- c) What is the output variable? What does it represent in this context?
- d) What is the y -intercept of $R(p)$? Explain the meaning of the y -intercept in the context of the given situation.
- e) What is the vertex of $R(p)$? Explain the meaning of the vertex in the context of the given situation.
- f) What are the p -intercept(s)? Explain what the p -intercept(s) means in the context of the given situation.
- g) Determine a reasonable domain of $R(p)$ in the context of the given situation.
- h) Determine a reasonable range of $R(p)$ in the context of the given situation.



5.7: Algebra Critique – Simplifying Rational Expressions

Learning Objectives

Together with your team:

- Critique the applicability of a mathematical approach or the validity of a mathematical conclusion.
- Simplify rational expressions.

A group of Algebra students were working on an ALEKS problems to prepare for the final exam. They are all working on different topics and want to make sure that their answers are valid before entering them into ALEKS. Your task is to decide whether each student's answer is valid or not. **If an answer is invalid, explain why and correct the student's work.**

1) The first student is working on the following ALEKS problem.

☰

● RATIONAL EXPRESSIONS

Simplifying a ratio of factored polynomials: Linear factors

Simplify.

$$\frac{4(2v+7)(v+8)}{16(v-4)(2v+7)}$$

You may leave the numerator and denominator of your answer in factored form.

The student's work:

$$\frac{4(2v+7)(v+8)}{16(v-4)(2v+7)} = \frac{4(2v+7)(v+8)}{16(v-4)(2v+7)}$$

$$\frac{4 \cdot 8}{16 \cdot -4} = \frac{1}{-2}$$

The student wants to enter in ALEKS

$$-\frac{1}{2}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

2) The second student is working on the following ALEKS problem.

☰

● RATIONAL EXPRESSIONS

Multiplying rational expressions made up of linear expressions

Multiply.

$$\frac{8x-56}{30x-25} \cdot \frac{6x-5}{4x-28}$$

[Simplify](#) your answer as much as possible.

The student's work:

$$\frac{8x-56}{30x-25} \cdot \frac{6x-5}{4x-28}$$

$$\frac{8(\cancel{x-7})}{5(\cancel{6x-5})} \cdot \frac{\cancel{6x-5}}{4(\cancel{x-7})}$$

$$\frac{8}{5} \cdot \frac{1}{4} = \frac{2}{5}$$

The student wants to enter in ALEKS:

$$\frac{2}{5}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

3) The third student is working on the following ALEKS problem.

≡

● RATIONAL EXPRESSIONS
 Dividing rational expressions involving linear expressions

Divide.

$$\frac{3}{2x+16} \div \frac{7}{4x+32}$$

Simplify your answer as much as possible.

The student's work:

$$\frac{3}{2x+16} \cdot \frac{4x+32}{7}$$

$$= \frac{12x+96}{14x+112}$$

The student wants to enter in ALEKS:

$$\frac{12x+96}{14x+112}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.

4) The fourth student is working on the following ALEKS problem:

≡

● RATIONAL EXPRESSIONS
 Adding rational expressions with linear denominators

Add.

$$\frac{3}{x-6} + \frac{4}{x+4}$$

Simplify your answer as much as possible.

The student's work:

$$\frac{3}{x-6} + \frac{4}{x+4}$$

$$= \frac{7}{2x-2}$$

The student wants to enter in ALEKS:

$$\frac{7}{2x-2}$$

a) The student's answer is: VALID INVALID

b) If their answer is INVALID, correct the student's work.